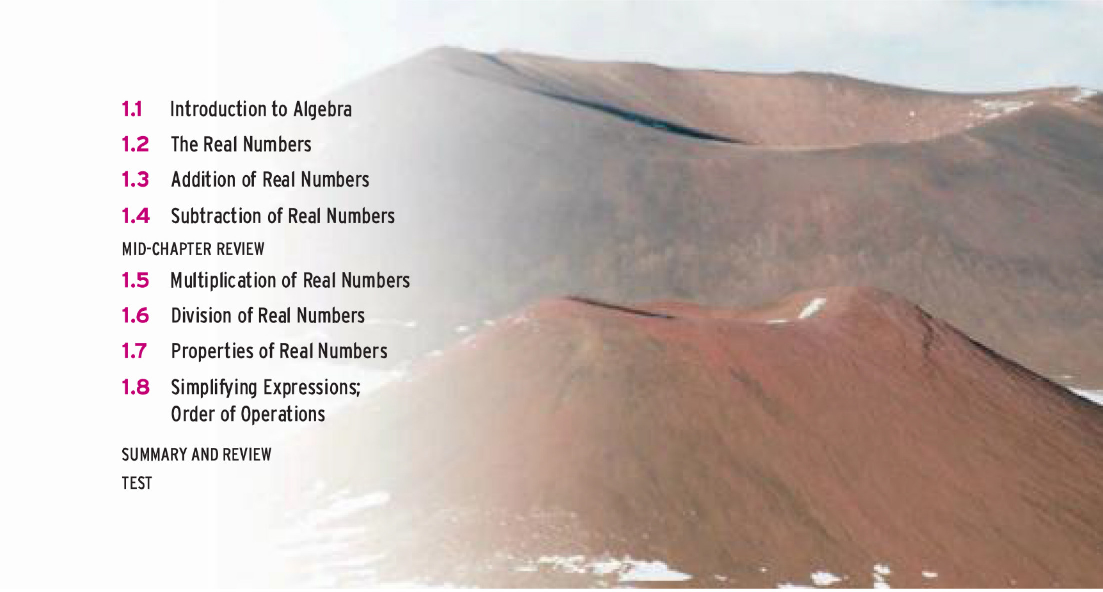
Introduction to Real Numbers and Algebraic Expressions

CHAPTER 1



Real-World Application

The tallest mountain in the world, when measured from base to peak, is Mauna Kea (White Mountain) in Hawaii. From its base 19,684 ft below sea level in the Hawaiian Trough, it rises 33,480 ft. What is the elevation of the peak above sea level?

Source: The Guinness Book of Records

This problem appears as Exercise 71 in Exercise Set 1.3.

1.1

Introduction to Algebra

OBJECTIVES

- **a** Evaluate algebraic expressions by substitution.
- **b** Translate phrases to algebraic expressions.

The study of algebra involves the use of equations to solve problems. Equations are constructed from algebraic expressions. The purpose of this section is to introduce you to the types of expressions encountered in algebra.

a Evaluating Algebraic Expressions

In arithmetic, you have worked with expressions such as

$$49 + 75, \qquad 8 \times 6.07, \qquad 29 - 14, \quad \text{and} \quad \frac{5}{6}.$$

In algebra, we can use letters to represent numbers and work with *algebraic expressions* such as

$$x + 75, \qquad 8 \times y, \qquad 29 - t, \quad \text{and} \quad \frac{a}{b}.$$

Sometimes a letter can represent various numbers. In that case, we call the letter a **variable**. Let $a =$ your age. Then a is a variable since a changes from year to year. Sometimes a letter can stand for just one number. In that case, we call the letter a **constant**. Let $b =$ your date of birth. Then b is a constant.

Where do algebraic expressions occur? Most often we encounter them when we are solving applied problems. For example, consider the bar graph shown at left, one that we might find in a book or a magazine. Suppose we want to know how much higher Mt. McKinley is than Mt. Evans. Using arithmetic, we might simply subtract. But let's see how we can determine this using algebra. We translate the problem into a statement of equality, an equation. It could be done as follows:

$$\begin{array}{ccccc} \text{Height of} & & \text{How much} & & \text{Height of} \\ \underbrace{\text{Mt. Evans}} & \text{plus} & \underbrace{\text{more}} & \text{is} & \underbrace{\text{Mt. McKinley}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 14{,}264 & + & x & = & 20{,}320. \end{array}$$

Note that we have an algebraic expression, $14{,}264 + x$, on the left of the equals sign. To find the number x, we can subtract 14,264 on both sides of the equation:

$$\begin{aligned} 14{,}264 + x &= 20{,}320 \\ 14{,}264 + x - 14{,}264 &= 20{,}320 - 14{,}264 \\ x &= 6056. \end{aligned}$$

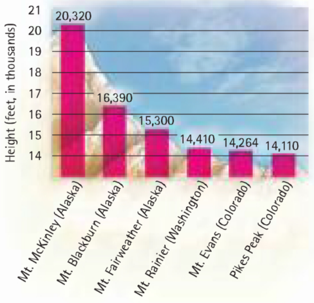
This value of x gives the answer, 6056 ft.

We call $14{,}264 + x$ an *algebraic expression* and $14{,}264 + x = 20{,}320$ an *algebraic equation*. Note that there is no equals sign, =, in an algebraic expression.

In arithmetic, you probably would do this subtraction without ever considering an equation. *In algebra, more complex problems are difficult to solve without first writing an equation.*

Do Exercise 1.

Mountain Peaks in the United States



SOURCE: U.S. Department of the Interior, Geological Survey

1. Translate this problem to an equation. Then solve the equation.

Mountain Peaks. There are 92 mountain peaks in the United States that are higher than 14,000 ft. The bar graph above shows data for six of these. How much higher is Mt. Fairweather than Mt. Rainier?

Answer

1. $14{,}410 + x = 15{,}300$; 890 ft

An **algebraic expression** consists of variables, constants, numerals, operation signs, and/or grouping symbols. When we replace a variable with a number, we say that we are **substituting** for the variable. When we replace all of the variables in an expression with numbers and carry out the operations in the expression, we are **evaluating the expression**.

EXAMPLE 1 Evaluate $x + y$ when $x = 37$ and $y = 29$.

We substitute 37 for x and 29 for y and carry out the addition:

$$x + y = 37 + 29 = 66.$$

The number 66 is called the **value** of the expression when $x = 37$ and $y = 29$.

Algebraic expressions involving multiplication can be written in several ways. For example, "8 times a" can be written as

$$8 \times a, \quad 8 \cdot a, \quad 8(a), \quad \text{or simply} \quad 8a.$$

Two letters written together without an operation symbol, such as ab, also indicate a multiplication.

EXAMPLE 2 Evaluate $3y$ when $y = 14$.

$$3y = 3(14) = 42$$

Do Exercises 2–4.

2. Evaluate $a + b$ when $a = 38$ and $b = 26$.

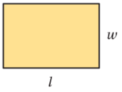
3. Evaluate $x - y$ when $x = 57$ and $y = 29$.

4. Evaluate $4t$ when $t = 15$.

EXAMPLE 3 ***Area of a Rectangle.*** The area A of a rectangle of length l and width w is given by the formula $A = lw$. Find the area when l is 24.5 in. and w is 16 in.

We substitute 24.5 in. for l and 16 in. for w and carry out the multiplication:

$$\begin{aligned} A = lw &= (24.5\text{ in.})(16\text{ in.}) \\ &= (24.5)(16)(\text{in.})(\text{in.}) \\ &= 392\text{ in}^2\text{, or 392 square inches.} \end{aligned}$$



Do Exercise 5.

5. Find the area of a rectangle when l is 24 ft and w is 8 ft.

Algebraic expressions involving division can also be written in several ways. For example, "8 divided by t" can be written as

$$8 \div t, \quad \frac{8}{t}, \quad 8/t, \quad \text{or} \quad 8 \cdot \frac{1}{t},$$

where the fraction bar is a division symbol.

EXAMPLE 4 Evaluate $\frac{a}{b}$ when $a = 63$ and $b = 9$.

We substitute 63 for a and 9 for b and carry out the division:

$$\frac{a}{b} = \frac{63}{9} = 7.$$

EXAMPLE 5 Evaluate $\frac{12m}{n}$ when $m = 8$ and $n = 16$.

$$\frac{12m}{n} = \frac{12 \cdot 8}{16} = \frac{96}{16} = 6$$

Answers

2. 64 3. 28 4. 60 5. 192 ft^2

6. Evaluate a/b when $a = 200$ and $b = 8$.

7. Evaluate $10p/q$ when $p = 40$ and $q = 25$.

8. Motorcycle Travel. Find the time it takes to travel 660 mi if the speed is 55 mph.

To the student: At the front of the text, you will find a Student Organizer card. This pullout card will help you keep track of important dates and useful contact information. You can also use it to plan time for class, study, work, and relaxation. By managing your time wisely, you will provide yourself the best possible opportunity to be successful in this course.

Answers

6. 25 **7.** 16 **8.** 12 hr

Do Exercises 6 and 7.

EXAMPLE 6 *Motorcycle Travel.* Ed wants to travel 660 mi on his motorcycle on a particular day. The time t, in hours, that it takes to travel 660 mi is given by

$$t = \frac{660}{r},$$

where r is the speed of Ed's motorcycle. Find the time of travel if the speed r is 60 mph.

We substitute 60 for r and carry out the division:

$$t = \frac{660}{r} = \frac{660}{60} = 11 \text{ hr}.$$

Do Exercise 8.

b Translating to Algebraic Expressions

In algebra, we translate problems to equations. The different parts of an equation are translations of word phrases to algebraic expressions. It is easier to translate if we know that certain words often translate to certain operation symbols.

KEY WORDS, PHRASES, AND CONCEPTS

ADDITION (+)	SUBTRACTION (−)	MULTIPLICATION (·)	DIVISION (÷)
add	subtract	multiply	divide
added to	subtracted from	multiplied by	divided by
sum	difference	product	quotient
total	minus	times	
plus	less than	of	
more than	decreased by		
increased by	take away		

EXAMPLE 7 Translate to an algebraic expression:

Twice (or two times) some number.

Think of some number, say, 8. We can write 2 times 8 as 2×8, or $2 \cdot 8$. We multiplied by 2. Do the same thing using a variable. We can use any variable we wish, such as x, y, m, or n. Let's use y to stand for some number. If we multiply by 2, we get an expression

$$y \times 2, \quad 2 \times y, \quad 2 \cdot y, \quad \text{or} \quad 2y.$$

In algebra, $2y$ is the expression generally used.

EXAMPLE 8 Translate to an algebraic expression:

Thirty-eight percent of some number.

Let $n =$ the number. The word "of" translates to a multiplication symbol, so we could write any of the following expressions as a translation:

$$38\% \cdot n, \quad 0.38 \times n, \quad \text{or} \quad 0.38n.$$

EXAMPLE 9 Translate to an algebraic expression:

Seven less than some number.

We let x represent the number. If the number were 10, then 7 less than 10 is $10 - 7$, or 3. If we knew the number to be 34, then 7 less than the number would be $34 - 7$. Thus if the number is x, then the translation is

$$x - 7.$$

Caution!

Note that $7 - x$ is *not* a correct translation of the expression in Example 9. The expression $7 - x$ is a translation of "seven minus some number" or "some number less than seven."

EXAMPLE 10 Translate to an algebraic expression:

Eighteen more than a number.

We let $t =$ the number. Now if the number were 6, then the translation would be $6 + 18$, or $18 + 6$. If we knew the number to be 17, then the translation would be $17 + 18$, or $18 + 17$. If the number is t, then the translation is

$$t + 18, \quad \text{or} \quad 18 + t.$$

EXAMPLE 11 Translate to an algebraic expression:

A number divided by 5.

We let $m =$ the number. Now if the number were 7, then the translation would be $7 \div 5$, or $7/5$, or $\frac{7}{5}$. If the number were 21, then the translation would be $21 \div 5$, or $21/5$, or $\frac{21}{5}$. If the number is m, then the translation is

$$m \div 5, \qquad m/5, \quad \text{or} \quad \frac{m}{5}.$$

EXAMPLE 12 Translate each phrase to an algebraic expression.

PHRASE	ALGEBRAIC EXPRESSION
Five more than some number	$n + 5$, or $5 + n$
Half of a number	$\frac{1}{2}t$, $\frac{t}{2}$, or $t/2$
Five more than three times some number	$3p + 5$, or $5 + 3p$
The difference of two numbers	$x - y$
Six less than the product of two numbers	$mn - 6$
Seventy-six percent of some number	$76\%z$, or $0.76z$
Four less than twice some number	$2x - 4$

Do Exercises 9–17.

Translate each phrase to an algebraic expression.

9. Eight less than some number

10. Eight more than some number

11. Four less than some number

12. Half of some number

13. Six more than eight times some number

14. The difference of two numbers

15. Fifty-nine percent of some number

16. Two hundred less than the product of two numbers

17. The sum of two numbers

Answers

9. $x - 8$ **10.** $y + 8$, or $8 + y$
11. $m - 4$ **12.** $\frac{1}{2} \cdot p$, or $\frac{p}{2}$
13. $8x + 6$, or $6 + 8x$ **14.** $a - b$
15. $59\%x$, or $0.59x$ **16.** $xy - 200$
17. $p + q$

1.1 Exercise Set

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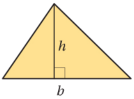
a Substitute to find values of the expressions in each of the following applied problems.

1. ***Commuting Time.*** It takes Erin 24 min less time to commute to work than it does George. Suppose that the variable x stands for the time it takes George to get to work. Then $x - 24$ stands for the time it takes Erin to get to work. How long does it take Erin to get to work if it takes George 56 min? 93 min? 105 min?

3. ***Area of a Triangle.*** The area A of a triangle with base b and height h is given by $A = \frac{1}{2}bh$. Find the area when $b = 45$ m (meters) and $h = 86$ m.



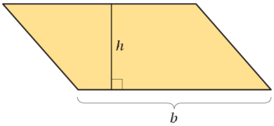
5. ***Distance Traveled.*** A driver who drives at a constant speed of r miles per hour for t hours will travel a distance of d miles given by $d = rt$ miles. How far will a driver travel at a speed of 65 mph for 4 hr?

7. ***Hockey Goal.*** The front of a regulation hockey goal is a rectangle that is 6 ft wide and 4 ft high. Find its area.

Source: National Hockey League

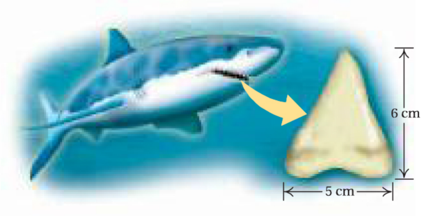
2. ***Enrollment Costs.*** At Emmett Community College, it costs $600 to enroll in the 8 A.M. section of Elementary Algebra. Suppose that the variable n stands for the number of students who enroll. Then $600n$ stands for the total amount of money collected for this course. How much is collected if 34 students enroll? 78 students? 250 students?

4. ***Area of a Parallelogram.*** The area A of a parallelogram with base b and height h is given by $A = bh$. Find the area of the parallelogram when the height is 15.4 cm (centimeters) and the base is 6.5 cm.



6. ***Simple Interest.*** The simple interest I on a principal of P dollars at interest rate r for time t, in years, is given by $I = Prt$. Find the simple interest on a principal of $4800 at 9% for 2 years. (*Hint*: $9\% = 0.09$.)

8. ***Zoology.*** A great white shark has triangular teeth. Each tooth measures about 5 cm across the base and has a height of 6 cm. Find the surface area of one side of one tooth. (See Exercise 3.)



Evaluate.

9. $8x$, when $x = 7$

10. $6y$, when $y = 7$

11. $\frac{c}{d}$, when $c = 24$ and $d = 3$

12. $\frac{p}{q}$, when $p = 16$ and $q = 2$

13. $\frac{3p}{q}$, when $p = 2$ and $q = 6$

14. $\frac{5y}{z}$, when $y = 15$ and $z = 25$

15. $\frac{x + y}{5}$, when $x = 10$ and $y = 20$

16. $\frac{p + q}{2}$, when $p = 2$ and $q = 16$

17. $\frac{x - y}{8}$, when $x = 20$ and $y = 4$

18. $\frac{m - n}{5}$, when $m = 16$ and $n = 6$

b Translate each phrase to an algebraic expression. Use any letter for the variable(s) unless directed otherwise.

19. Seven more than some number

20. Nine more than some number

21. Twelve less than some number

22. Fourteen less than some number

23. Some number increased by four

24. Some number increased by thirteen

25. b more than a

26. c more than d

27. x divided by y

28. c divided by h

29. x plus w

30. s added to t

31. m subtracted from n

32. p subtracted from q

33. The sum of two numbers

34. The sum of nine and some number

35. Twice some number

36. Three times some number

37. Three multiplied by some number

38. The product of eight and some number

39. Six more than four times some number

40. Two more than six times some number

41. Eight less than the product of two numbers

42. The product of two numbers minus seven

43. Five less than twice some number

44. Six less than seven times some number

45. Three times some number plus eleven

46. Some number times 8 plus 5

47. The sum of four times a number plus three times another number

48. Five times a number minus eight times another number

49. The product of 89% and your salary

50. 67% of the women attending

51. Your salary after a 5% salary increase if your salary before the increase was *s*

52. The price of a blouse after a 30% reduction if the price before the reduction was *P*

53. Danielle drove at a speed of 65 mph for *t* hours. How far did Danielle travel? (See Exercise 5.)

54. Dino drove his pickup truck at 55 mph for *t* hours. How far did he travel? (See Exercise 5.)

55. Lisa had $50 before spending *x* dollars on pizza. How much money remains?

56. Juan has *d* dollars before spending $29.95 on a DVD of the movie *Chicago*. How much did Juan have after the purchase?

57. Robert's part-time job pays $8.50 per hour. How much does he earn for working *n* hours?

58. Meredith pays her babysitter $10 per hour. What does it cost her to hire the sitter for *m* hours?

Synthesis

To the student and the instructor: The Synthesis exercises found at the end of most exercise sets challenge students to combine concepts or skills studied in that section or in preceding parts of the text.

Evaluate.

59. $\frac{a - 2b + c}{4b - a}$, when $a = 20, b = 10$, and $c = 5$

60. $\frac{x}{y} - \frac{5}{x} + \frac{2}{y}$, when $x = 30$ and $y = 6$

61. $\frac{12 - c}{c + 12b}$, when $b = 1$ and $c = 12$

62. $\frac{2w - 3z}{7y}$, when $w = 5, y = 6$, and $z = 1$

1.2

The Real Numbers

A **set** is a collection of objects. For our purposes, we will most often be considering sets of numbers. One way to name a set uses what is called **roster notation**. For example, roster notation for the set containing the numbers 0, 2, and 5 is $\{0, 2, 5\}$.

Sets that are part of other sets are called **subsets**. In this section, we become acquainted with the set of *real numbers* and its various subsets.

Two important subsets of the real numbers are listed below using roster notation.

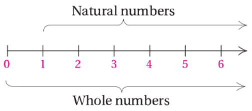
NATURAL NUMBERS

The set of **natural numbers** $= \{1, 2, 3, \ldots\}$. These are the numbers used for counting.

WHOLE NUMBERS

The set of **whole numbers** $= \{0, 1, 2, 3, \ldots\}$. This is the set of natural numbers and 0.

We can represent these sets on the number line. The natural numbers are to the right of zero. The whole numbers are the natural numbers and zero.



We create a new set, called the *integers*, by starting with the whole numbers, 0, 1, 2, 3, and so on. For each natural number 1, 2, 3, and so on, we obtain a new number to the left of zero on the number line:

For the number 1, there will be an *opposite* number -1 (negative 1).

For the number 2, there will be an *opposite* number -2 (negative 2).

For the number 3, there will be an *opposite* number -3 (negative 3), and so on.

The **integers** consist of the whole numbers and these new numbers.

INTEGERS

The set of **integers** $= \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$.

OBJECTIVES

a State the integer that corresponds to a real-world situation.

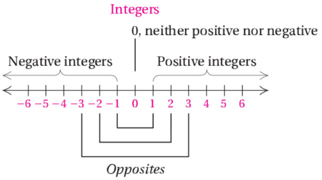
b Graph rational numbers on the number line.

c Convert from fraction notation for a rational number to decimal notation.

d Determine which of two real numbers is greater and indicate which, using $<$ or $>$. Given an inequality like $a > b$, write another inequality with the same meaning. Determine whether an inequality like $-3 \le 5$ is true or false.

e Find the absolute value of a real number.

We picture the integers on the number line as follows.

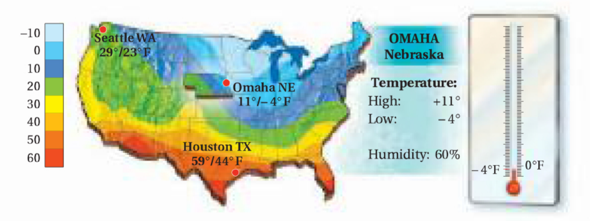


We call the integers to the left of zero **negative integers**. The natural numbers are also called **positive integers**. Zero is neither positive nor negative. We call −1 and 1 **opposites** of each other. Similarly, −2 and 2 are opposites, −3 and 3 are opposites, −100 and 100 are opposites, and 0 is its own opposite. Pairs of opposite numbers like −3 and 3 are the same distance from zero. The integers extend infinitely on the number line to the left and right of zero.

a Integers and the Real World

Integers correspond to many real-world problems and situations. The following examples will help you get ready to translate problem situations that involve integers to mathematical language.

EXAMPLE 1 Tell which integer corresponds to this situation: The temperature is 4 degrees below zero.



The integer −4 corresponds to the situation. The temperature is −4°.

EXAMPLE 2 *"Jeopardy."* Tell which integer corresponds to this situation: A contestant missed a $600 question on the television game show "Jeopardy."

Missing a $600 question means −600.

Missing a $600 question causes a $600 loss on the score—that is, the contestant earns −600 dollars.

EXAMPLE 3 *Elevation.* Tell which integer corresponds to this situation: The shores of California's largest lake, the Salton Sea, are 227 ft below sea level.
Source: Salton Sea Authority

The integer -227 corresponds to the situation. The elevation is -227 ft.

EXAMPLE 4 *Stock Price Change.* Tell which integers correspond to this situation: Hal owns a stock whose price decreased \$16 per share over a recent period. He owns another stock whose price increased \$2 per share over the same period.

The integer -16 corresponds to the decrease in the value of the first stock. The integer 2 represents the increase in the value of the second stock.

Do Exercises 1–5.

Tell which integers correspond to each situation.

1. **Temperature High and Low.** The highest recorded temperature in Nevada is 125°F on June 29, 1994, in Laughlin. The lowest recorded temperature in Nevada is 50°F below zero on January 8, 1937, in San Jacinto.
Source: National Climatic Data Center, Asheville, NC, and Storm Phillips, STORMFAX, INC.

2. **Stock Decrease.** The price of a stock decreased \$3 per share over a recent period.

3. At 10 sec before liftoff, ignition occurs. At 148 sec after liftoff, the first stage is detached from the rocket.

4. The halfback gained 8 yd on first down. The quarterback was sacked for a 5-yd loss on second down.

5. A submarine dove 120 ft, rose 50 ft, and then dove 80 ft.

b The Rational Numbers

We created the set of integers by obtaining a negative number for each natural number and also including 0. To create a larger number system, called the set of **rational numbers**, we consider quotients of integers with nonzero divisors. The following are some examples of rational numbers:

$$\frac{2}{3},\quad -\frac{2}{3},\quad \frac{7}{1},\quad 4,\quad -3,\quad 0,\quad \frac{23}{-8},\quad 2.4,\quad -0.17,\quad 10\frac{1}{2}.$$

The number $-\frac{2}{3}$ (read "negative two-thirds") can also be named $\frac{-2}{3}$ or $\frac{2}{-3}$; that is,

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}.$$

The number 2.4 can be named $\frac{24}{10}$ or $\frac{12}{5}$, and -0.17 can be named $-\frac{17}{100}$. We can describe the set of rational numbers as follows.

RATIONAL NUMBERS

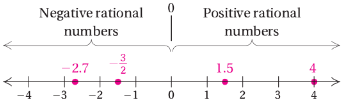
The set of **rational numbers** = the set of numbers $\frac{a}{b}$, where a and b are integers and b is not equal to 0 ($b \neq 0$).

Answers

1. 125; -50 2. The integer -3 corresponds to the decrease in the stock's value.
3. -10; 148 4. 8; -5 5. -120; 50; -80

Note that this new set of numbers, the rational numbers, contains the whole numbers, the integers, the arithmetic numbers (also called the nonnegative rational numbers), and the negative rational numbers.

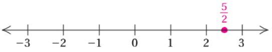
We picture the rational numbers on the number line as follows.



To **graph** a number means to find and mark its point on the number line. Some rational numbers are graphed in the preceding figure.

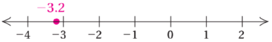
EXAMPLE 5 Graph: $\frac{5}{2}$.

The number $\frac{5}{2}$ can also be named $2\frac{1}{2}$, or 2.5. Its graph is halfway between 2 and 3.



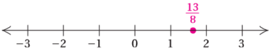
EXAMPLE 6 Graph: −3.2.

The graph of −3.2 is $\frac{2}{10}$ of the way from −3 to −4.



EXAMPLE 7 Graph: $\frac{13}{8}$.

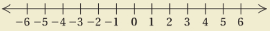
The number $\frac{13}{8}$ can also be named $1\frac{5}{8}$, or 1.625. The graph is $\frac{5}{8}$ of the way from 1 to 2.



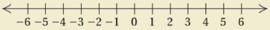
Do Exercises 6–8.

Graph on the number line.

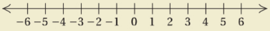
6. $-\frac{7}{2}$



7. 1.4



8. $-\frac{11}{4}$



c Notation for Rational Numbers

Each rational number can be named using fraction notation or decimal notation.

EXAMPLE 8 Convert to decimal notation: $-\frac{5}{8}$.

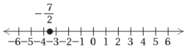
We first find decimal notation for $\frac{5}{8}$. Since $\frac{5}{8}$ means 5 ÷ 8, we divide.

$$\begin{array}{r} 0.6\,2\,5 \\ 8\,\overline{)\,5.0\,0\,0} \\ \underline{4\,8} \\ 2\,0 \\ \underline{1\,6} \\ 4\,0 \\ \underline{4\,0} \\ 0 \end{array}$$

Thus, $\frac{5}{8} = 0.625$, so $-\frac{5}{8} = -0.625$.

Answers

6.



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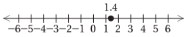

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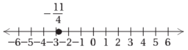
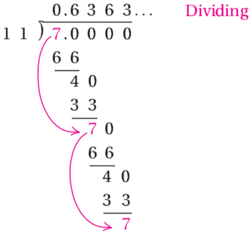


Decimal notation for $-\frac{5}{8}$ is -0.625. We consider -0.625 to be a **terminating decimal**. Decimal notation for some numbers repeats.

EXAMPLE 9 Convert to decimal notation: $\frac{7}{11}$.



We can abbreviate **repeating decimal** notation by writing a bar over the repeating part—in this case, we write $0.\overline{63}$. Thus, $\frac{7}{11} = 0.\overline{63}$.

Each rational number can be expressed in either terminating or repeating decimal notation.

The following are other examples showing how rational numbers can be named using fraction notation or decimal notation:

$$0 = \frac{0}{8}, \qquad \frac{27}{100} = 0.27, \qquad -8\frac{3}{4} = -8.75, \qquad -\frac{13}{6} = -2.1\overline{6}.$$

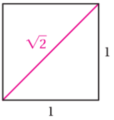
Do Exercises 9–11.

d The Real Numbers and Order

Every rational number has a point on the number line. However, there are some points on the line for which there is no rational number. These points correspond to what are called **irrational numbers**.

What kinds of numbers are irrational? One example is the number π, which is used in finding the area and the circumference of a circle: $A = \pi r^2$ and $C = 2\pi r$.

Another example of an irrational number is the square root of 2, named $\sqrt{2}$. It is the length of the diagonal of a square with sides of length 1. It is also the number that when multiplied by itself gives 2—that is, $\sqrt{2} \cdot \sqrt{2} = 2$. There is no rational number that can be multiplied by itself to get 2. But the following are rational *approximations*:



1.4 is an approximation of $\sqrt{2}$ because $(1.4)^2 = 1.96$;

1.41 is a better approximation because $(1.41)^2 = 1.9881$;

1.4142 is an even better approximation because $(1.4142)^2 = 1.99996164$.

We can find rational approximations for square roots using a calculator.

Find decimal notation.

9. $-\frac{3}{8}$

10. $-\frac{6}{11}$

11. $\frac{4}{3}$

Calculator Corner

Approximating Square Roots and π Square roots are found by pressing 2ND √. (√ is the second operation associated with the x^2 key.)

To find an approximation for $\sqrt{48}$, we press 2ND √ 4 8 ENTER. The approximation 6.92820323 is displayed.

To find $8 \cdot \sqrt{13}$, we press 8 2ND √ 1 3 ENTER. The approximation 28.8444102 is displayed.

The number π is used widely enough to have its own key. (π is the second operation associated with the ^ key.)

To approximate π, we press 2ND π ENTER. The approximation 3.141592654 is displayed.

Exercises: Approximate.

1. $\sqrt{76}$

2. $\sqrt{317}$

3. $15 \cdot \sqrt{20}$

4. $29 + \sqrt{42}$

5. π

6. $29 \cdot \pi$

7. $\pi \cdot 13^2$

8. $5 \cdot \pi + 8 \cdot \sqrt{237}$

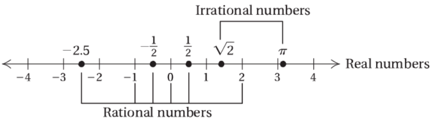
Answers

9. -0.375 **10.** $-0.\overline{54}$ **11.** $1.\overline{3}$

Decimal notation for rational numbers *either* terminates *or* repeats.
Decimal notation for irrational numbers *neither* terminates *nor* repeats.

Some other examples of irrational numbers are $\sqrt{3}$, $-\sqrt{8}$, $\sqrt{11}$, and 0.121221222122221. . . . Whenever we take the square root of a number that is not a perfect square, we will get an irrational number.

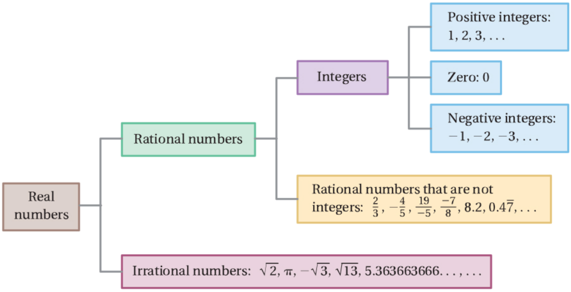
The rational numbers and the irrational numbers together correspond to all the points on the number line and make up what is called the **real-number system**.



REAL NUMBERS

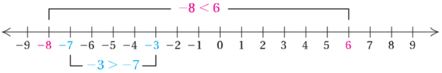
The set of **real numbers** = The set of all numbers corresponding to points on the number line.

The real numbers consist of the rational numbers and the irrational numbers. The following figure shows the relationships among various kinds of numbers.



Order

Real numbers are named in order on the number line, increasing as we move from left to right. For any two numbers on the line, the one on the left is less than the one on the right.



We use the symbol **<** to mean "**is less than**." The sentence $-8 < 6$ means "-8 is less than 6." The symbol **>** means "**is greater than**." The sentence $-3 > -7$ means "-3 is greater than -7." The sentences $-8 < 6$ and $-3 > -7$ are **inequalities**.

Calculator Corner

Negative Numbers on a Calculator; Converting to Decimal Notation We use the opposite key (−) to enter negative numbers on a graphing calculator. Note that this is different from the − key, which is used for the operation of subtraction. To convert $-\frac{5}{8}$ to decimal notation, as in Example 8, we press (−) 5 ÷ 8 ENTER. The result is -0.625.

```
-5/8
                -.625
```

Exercises: Convert each of the following negative numbers to decimal notation.

1. $-\frac{3}{4}$

2. $-\frac{9}{20}$

3. $-\frac{1}{8}$

4. $-\frac{9}{5}$

5. $-\frac{27}{40}$

6. $-\frac{11}{16}$

7. $-\frac{7}{2}$

8. $-\frac{19}{25}$

EXAMPLES Use either $<$ or $>$ for $\square$ to write a true sentence.

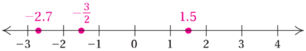
10. $2 \square 9$ Since 2 is to the left of 9, 2 is less than 9, so $2 < 9$.

11. $-7 \square 3$ Since -7 is to the left of 3, we have $-7 < 3$.

12. $6 \square -12$ Since 6 is to the right of -12, then $6 > -12$.

13. $-18 \square -5$ Since -18 is to the left of -5, we have $-18 < -5$.

14. $-2.7 \square -\frac{3}{2}$ The answer is $-2.7 < -\frac{3}{2}$.



15. $1.5 \square -2.7$ The answer is $1.5 > -2.7$.

16. $1.38 \square 1.83$ The answer is $1.38 < 1.83$.

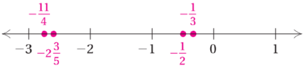
17. $-3.45 \square 1.32$ The answer is $-3.45 < 1.32$.

18. $-4 \square 0$ The answer is $-4 < 0$.

19. $5.8 \square 0$ The answer is $5.8 > 0$.

20. $\frac{5}{8} \square \frac{7}{11}$ We convert to decimal notation: $\frac{5}{8} = 0.625$ and $\frac{7}{11} = 0.6363\ldots$. Thus, $\frac{5}{8} < \frac{7}{11}$.

21. $-\frac{1}{2} \square -\frac{1}{3}$ The answer is $-\frac{1}{2} < -\frac{1}{3}$.



22. $-2\frac{3}{5} \square -\frac{11}{4}$ The answer is $-2\frac{3}{5} > -\frac{11}{4}$.

Do Exercises 12–19.

Use either $<$ or $>$ for $\square$ to write a true sentence.

12. $-3 \square 7$

13. $-8 \square -5$

14. $7 \square -10$

15. $3.1 \square -9.5$

16. $-4.78 \square -5.01$

17. $-\frac{2}{3} \square -\frac{5}{9}$

18. $-\frac{11}{8} \square \frac{23}{15}$

19. $0 \square -9.9$

Note that both $-8 < 6$ and $6 > -8$ are true. Every true inequality yields another true inequality when we interchange the numbers or variables and reverse the direction of the inequality sign.

ORDER; >, <

$a < b$ also has the meaning $b > a$.

EXAMPLES Write another inequality with the same meaning.

23. $-3 > -8$ The inequality $-8 < -3$ has the same meaning.

24. $a < -5$ The inequality $-5 > a$ has the same meaning.

A helpful mental device is to think of an inequality sign as an "arrow" with the arrowhead pointing to the smaller number.

Do Exercises 20 and 21.

Write another inequality with the same meaning.

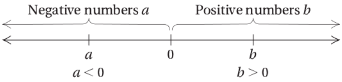
20. $-5 < 7$

21. $x > 4$

Answers

12. $<$ **13.** $<$ **14.** $>$ **15.** $>$ **16.** $>$
17. $<$ **18.** $<$ **19.** $>$ **20.** $7 > -5$
21. $4 < x$

Note that all positive real numbers are greater than zero and all negative real numbers are less than zero.



If b is a positive real number, then $b > 0$.
If a is a negative real number, then $a < 0$.

Expressions like $a \leq b$ and $b \geq a$ are also inequalities. We read $\boldsymbol{a \leq b}$ as "***a* is less than or equal to *b***." We read $\boldsymbol{a \geq b}$ as "***a* is greater than or equal to *b***."

EXAMPLES Write true or false for each statement.

25. $-3 \leq 5.4$ True since $-3 < 5.4$ is true

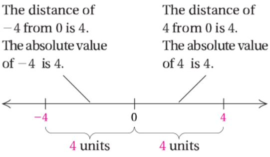
26. $-3 \leq -3$ True since $-3 = -3$ is true

27. $-5 \geq 1\frac{2}{3}$ False since neither $-5 > 1\frac{2}{3}$ nor $-5 = 1\frac{2}{3}$ is true

Do Exercises 22–24.

e Absolute Value

From the number line, we see that numbers like 4 and -4 are the same distance from zero. Distance is always a nonnegative number. We call the distance of a number from zero on the number line the **absolute value** of the number.



ABSOLUTE VALUE

The **absolute value** of a number is its distance from zero on the number line. We use the symbol $|x|$ to represent the absolute value of a number x.

Write true or false.

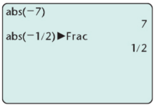
22. $-4 \leq -6$

23. $7.8 \geq 7.8$

24. $-2 \leq \frac{3}{8}$

Calculator Corner

Absolute Value The absolute-value operation is the first item in the Catalog on the TI-84 Plus graphing calculator. To find $|-7|$, as in Example 28 on the following page, we first press 2ND CATALOG ENTER to copy "abs(" to the home screen. (CATALOG is the second operation associated with the 0 numeric key.) Then we press (−) 7) ENTER. The result is 7. To find $|-\frac{1}{2}|$ and express the result as a fraction, we press 2ND CATALOG ENTER (−) 1 ÷ 2) MATH 1 ENTER. The result is $\frac{1}{2}$.



```
abs(−7)
                7
abs(−1/2)►Frac
              1/2
```

Exercises: Find the absolute value.

1. $|-5|$

2. $|17|$

3. $|0|$

4. $|6.48|$

5. $|-12.7|$

6. $|-0.9|$

7. $\left|-\frac{5}{7}\right|$

8. $\left|\frac{4}{3}\right|$

Answers

22. False 23. True 24. True

FINDING ABSOLUTE VALUE

a) If a number is negative, its absolute value is its opposite.

b) If a number is positive or zero, its absolute value is the same as the number.

EXAMPLES Find the absolute value.

28. $|-7|$ The distance of -7 from 0 is 7, so $|-7| = 7$.

29. $|12|$ The distance of 12 from 0 is 12, so $|12| = 12$.

30. $|0|$ The distance of 0 from 0 is 0, so $|0| = 0$.

31. $\left|\frac{3}{2}\right| = \frac{3}{2}$

32. $|-2.73| = 2.73$

Do Exercises 25–28.

Find the absolute value.

25. $|8|$

26. $|-9|$

27. $\left|-\frac{2}{3}\right|$

28. $|5.6|$

STUDY TIPS

USING THIS TEXTBOOK

You will find many Study Tips throughout the book. An index of all Study Tips can be found on the Bittinger Student Organizer at the front of the book. One of the most important ways to improve your math study skills is to learn the proper use of the textbook. Here we highlight a few points that we consider most helpful.

- **Be sure to note the special symbols ⓐ, ⓑ, ⓒ, and so on, that correspond to the objectives you are to be able to master.** The first time you see them is in the margin at the beginning of each section; the second time is in the subheadings of each section; and the third time is in the exercise set for the section. You will also find them referred to in the skill maintenance exercises in each exercise set, in the mid-chapter review, and in the review exercises at the end of the chapter, as well as in the answers to the chapter tests and the cumulative reviews. These objective symbols allow you to refer to the appropriate place in the text whenever you need to review a topic.
- **Read and study each step of each example.** The examples include important side comments that explain each step. These carefully chosen examples and notes prepare you for success in the exercise set.
- **Stop and do the margin exercises as you study a section.** Doing the margin exercises is one of the most effective ways to enhance your ability to learn mathematics from this text. Don't deprive yourself of its benefits!
- **Note the icons listed at the top of each exercise set.** These refer to the many distinctive multimedia study aids that accompany the book.
- **Odd-numbered exercises.** Usually an instructor assigns some odd-numbered exercises. When you complete these, you can check your answers at the back of the book. If you miss any, check your work in the *Student's Solutions Manual* or ask your instructor for guidance.
- **Even-numbered exercises.** Whether or not your instructor assigns the even-numbered exercises, always do some on your own. Remember, there are no answers given for the class tests, so you need to practice doing exercises without answers. Check your answers later with a friend or your instructor.

Answers

25. 8 **26.** 9 **27.** $\frac{2}{3}$ **28.** 5.6

1.2 Exercise Set

For Extra Help

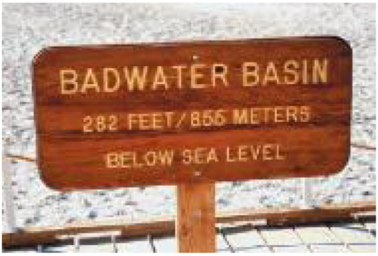
MyMathLab

MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

a State the integers that correspond to the situation.

1. ***Death Valley.*** With an elevation of 282 ft below sea level, Badwater Basin in California's Death Valley has the lowest elevation in the United States.

Source: Desert USA



2. ***Pollution Fine.*** The Massey Energy Company, the nation's fourth largest coal producer, was fined \$20 million for water pollution in 2008.

Source: Environmental Protection Agency

3. On Wednesday, the temperature was 24° above zero. On Thursday, it was 2° below zero.

4. A student deposited her tax refund of \$750 in a savings account. Two weeks later, she withdrew \$125 to pay technology fees.

5. ***Temperature Extremes.*** The highest temperature ever created in a lab is 3,600,000,000°F. The lowest temperature ever created is approximately 460°F below zero.

Sources: *Live Science; Guinness Book of World Records*

6. ***Extreme Climate.*** Verkhoyansk, a river port in northeast Siberia, has the most extreme climate on the planet. Its average monthly winter temperature is 58.5°F below zero, and its average monthly summer temperature is 56.5°F.

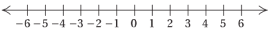
Source: *Guinness Book of World Records*

7. In bowling, the Alley Cats are 34 pins behind the Strikers going into the last frame. Describe the situation of each team.

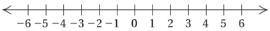
8. During a video game, Maggie intercepted a missile worth 20 points, lost a starship worth 150 points, and captured a landing base worth 300 points.

b Graph the number on the number line.

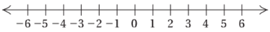
9. $\frac{10}{3}$



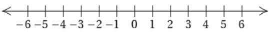
10. $-\frac{17}{4}$



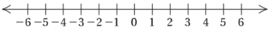
11. -5.2



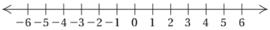
12. 4.78



13. $-4\frac{2}{5}$



14. $2\frac{6}{11}$



c Convert to decimal notation.

15. $-\frac{7}{8}$

16. $-\frac{3}{16}$

17. $\frac{5}{6}$

18. $\frac{5}{3}$

19. $-\frac{7}{6}$

20. $-\frac{5}{12}$

21. $\frac{2}{3}$

22. $-\frac{11}{9}$

23. $\frac{1}{10}$

24. $\frac{1}{4}$

25. $-\frac{1}{2}$

26. $\frac{9}{8}$

27. $\frac{4}{25}$

28. $-\frac{7}{20}$

d Use either $<$ or $>$ for $\square$ to write a true sentence.

29. $8 \square 0$

30. $3 \square 0$

31. $-8 \square 3$

32. $6 \square -6$

33. $-8 \square 8$

34. $0 \square -9$

35. $-8 \square -5$

36. $-4 \square -3$

37. $-5 \square -11$

38. $-3 \square -4$

39. $-6 \square -5$

40. $-10 \square -14$

41. $2.14 \square 1.24$

42. $-3.3 \square -2.2$

43. $-14.5 \square 0.011$

44. $17.2 \square -1.67$

45. $-12.88 \square -6.45$

46. $-14.34 \square -17.88$

47. $-\frac{1}{2} \square -\frac{2}{3}$

48. $-\frac{5}{4} \square -\frac{3}{4}$

49. $-\frac{2}{3} \square \frac{1}{3}$

50. $\frac{3}{4} \square -\frac{5}{4}$

51. $\frac{5}{12} \square \frac{11}{25}$

52. $-\frac{13}{16} \square -\frac{5}{9}$

Write an inequality with the same meaning.

53. $-6 > x$

54. $x < 8$

55. $-10 \leq y$

56. $12 \geq t$

Write true or false.

57. $-5 \leq -6$

58. $-7 \geq -10$

59. $4 \geq 4$

60. $7 \leq 7$

61. $-3 \geq -11$

62. $-1 \leq -5$

63. $0 \geq 8$

64. $-5 \leq 7$

e Find the absolute value.

65. $|-3|$

66. $|-6|$

67. $|10|$

68. $|11|$

69. $|0|$

70. $|-2.7|$

71. $|-30.4|$

72. $|325|$

73. $\left|-\frac{2}{3}\right|$

74. $\left|-\frac{10}{7}\right|$

75. $\left|\frac{0}{4}\right|$

76. $|14.8|$

77. $|-2.65|$

78. $\left|-3\frac{5}{8}\right|$

79. $\left|-7\frac{4}{5}\right|$

Skill Maintenance

This heading indicates that the exercises that follow are Skill Maintenance exercises, which review any skill previously studied in the text. You can expect such exercises in every exercise set. Answers to *all* skill maintenance exercises are found at the back of the book. If you miss an exercise, restudy the objective shown in red.

Evaluate. [1.1a]

80. $15y$, for $y = 7$

81. $\frac{36}{p}$, for $p = 4$

82. $w - s$, for $w = 23$ and $s = 10$

83. $5a - 2b$, for $a = 9$ and $b = 3$

84. $\frac{5c}{d}$, for $c = 15$ and $d = 25$

85. $\frac{2x + y}{3}$, for $x = 12$ and $y = 9$

86. $\frac{q - r}{8}$, for $q = 30$ and $r = 6$

87. $\frac{w}{4y}$, for $w = 52$ and $y = 13$

Synthesis

List in order from the least to the greatest.

88. $-\frac{2}{3}, \frac{1}{2}, -\frac{3}{4}, -\frac{5}{6}, \frac{3}{8}, \frac{1}{6}$

89. $\frac{2}{3}, -\frac{1}{7}, \frac{1}{3}, -\frac{2}{7}, -\frac{2}{3}, \frac{2}{5}, -\frac{1}{3}, -\frac{2}{5}, \frac{9}{8}$

90. $-5.16, -4.24, -8.76, 5.23, 1.85, -2.13$

91. $-8\frac{7}{8}, 7^1, -5, |-6|, 4, |3|, -8\frac{5}{8}, -100, 0, 1^7, \frac{14}{4}, -\frac{67}{8}$

Given that $0.\overline{3} = \frac{1}{3}$ and $0.\overline{6} = \frac{2}{3}$, express each of the following as a quotient or a ratio of two integers.

92. $0.\overline{1}$

93. $0.\overline{9}$

94. $5.\overline{5}$

1.3

Addition of Real Numbers

In this section, we consider addition of real numbers. First, to gain an understanding, we add using the number line. Then we consider rules for addition.

ADDITION ON THE NUMBER LINE

To do the addition $a + b$ on the number line, start at 0, move to a, and then move according to b.

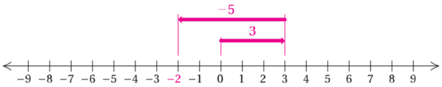
a) If b is positive, move from a to the right.

b) If b is negative, move from a to the left.

c) If b is 0, stay at a.

EXAMPLE 1 Add: $3 + (-5)$.

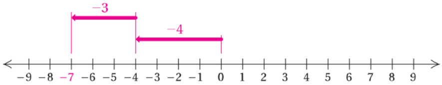
We start at 0 and move to 3. Then we move 5 units left since -5 is negative.



$3 + (-5) = -2$

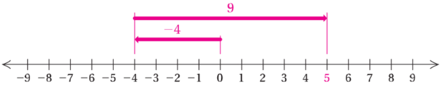
EXAMPLE 2 Add: $-4 + (-3)$.

We start at 0 and move to -4. Then we move 3 units left since -3 is negative.



$-4 + (-3) = -7$

EXAMPLE 3 Add: $-4 + 9$.



$-4 + 9 = 5$

OBJECTIVES

- **a** Add real numbers without using the number line.
- **b** Find the opposite, or additive inverse, of a real number.
- **c** Solve applied problems involving addition of real numbers.

SKILL TO REVIEW

Objective 1.1a: Evaluate algebraic expressions by substitution.

1. Evaluate $t - h$ when $t = 1$ and $h = 0.05$.

2. Evaluate $44 - 9q$ when $q = 3$.

STUDY TIPS

SMALL STEPS LEAD TO GREAT SUCCESS

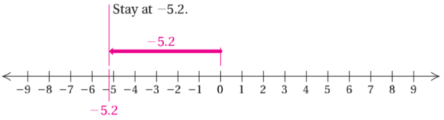
What is your long-term goal for getting an education? How does math help you to attain that goal? As you begin this course, approach each short-term task, such as going to class, asking questions, using your time wisely, and doing your homework, as part of the framework of your long-term goal.

Answers

Skill to Review:

1. 0.95 **2.** 17

EXAMPLE 4 Add: $-5.2 + 0$.



$-5.2 + 0 = -5.2$

Do Exercises 1–6.

a Adding Without the Number Line

You may have noticed some patterns in the preceding examples. These lead us to rules for adding without using the number line that are more efficient for adding larger numbers.

RULES FOR ADDITION OF REAL NUMBERS

1. *Positive numbers*: Add the same as arithmetic numbers. The answer is positive.
2. *Negative numbers*: Add absolute values. The answer is negative.
3. *A positive number and a negative number*:
   - If the numbers have the same absolute value, the answer is 0.
   - If the numbers have different absolute values, subtract the smaller absolute value from the larger. Then:
     a) If the positive number has the greater absolute value, the answer is positive.
     b) If the negative number has the greater absolute value, the answer is negative.
4. *One number is zero*: The sum is the other number.

Rule 4 is known as the **identity property of 0.** It says that for any real number a, $a + 0 = a$.

EXAMPLES Add without using the number line.

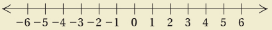
5. $-12 + (-7) = -19$ Two negatives. Add the absolute values: $|-12| + |-7| = 12 + 7 = 19$. Make the answer *negative*: -19.

6. $-1.4 + 8.5 = 7.1$ One negative, one positive. Find the absolute values: $|-1.4| = 1.4$; $|8.5| = 8.5$. Subtract the smaller absolute value from the larger: $8.5 - 1.4 = 7.1$. The *positive* number, 8.5, has the larger absolute value, so the answer is *positive*: 7.1.

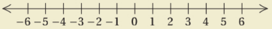
7. $-36 + 21 = -15$ One negative, one positive. Find the absolute values: $|-36| = 36$; $|21| = 21$. Subtract the smaller absolute value from the larger: $36 - 21 = 15$. The *negative* number, -36, has the larger absolute value, so the answer is *negative*: -15.

Add using the number line.

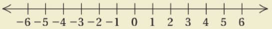
1. $0 + (-3)$



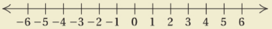
2. $1 + (-4)$



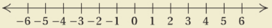
3. $-3 + (-2)$



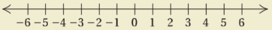
4. $-3 + 7$



5. $-2.4 + 2.4$



6. $-\frac{5}{2} + \frac{1}{2}$



Answers

1. -3 **2.** -3 **3.** -5
4. 4 **5.** 0 **6.** -2

8. $1.5 + (-1.5) = 0$ The numbers have the same absolute value. The sum is 0.

9. $-\frac{7}{8} + 0 = -\frac{7}{8}$ One number is zero. The sum is $-\frac{7}{8}$.

10. $-9.2 + 3.1 = -6.1$

11. $-\frac{3}{2} + \frac{9}{2} = \frac{6}{2} = 3$

12. $-\frac{2}{3} + \frac{5}{8} = -\frac{16}{24} + \frac{15}{24} = -\frac{1}{24}$

Do Exercises 7–20.

Add without using the number line.

7. $-5 + (-6)$

8. $-9 + (-3)$

9. $-4 + 6$

10. $-7 + 3$

11. $5 + (-7)$

12. $-20 + 20$

13. $-11 + (-11)$

14. $10 + (-7)$

15. $-0.17 + 0.7$

16. $-6.4 + 8.7$

17. $-4.5 + (-3.2)$

18. $-8.6 + 2.4$

19. $\frac{5}{9} + \left(-\frac{7}{9}\right)$

20. $-\frac{1}{5} + \left(-\frac{3}{4}\right)$

Suppose we want to add several numbers, some positive and some negative, as follows. How can we proceed?

$$15 + (-2) + 7 + 14 + (-5) + (-12)$$

We can change grouping and order as we please when adding. For instance, we can group the positive numbers together and the negative numbers together and add them separately. Then we add the two results.

EXAMPLE 13 Add: $15 + (-2) + 7 + 14 + (-5) + (-12)$.

a) $15 + 7 + 14 = 36$ Adding the positive numbers

b) $-2 + (-5) + (-12) = -19$ Adding the negative numbers

$36 + (-19) = 17$ Adding the results in (a) and (b)

We can also add the numbers in any other order we wish, say, from left to right as follows:

$$\begin{aligned} 15 + (-2) + 7 + 14 + (-5) + (-12) &= 13 + 7 + 14 + (-5) + (-12) \\ &= 20 + 14 + (-5) + (-12) \\ &= 34 + (-5) + (-12) \\ &= 29 + (-12) \\ &= 17 \end{aligned}$$

Do Exercises 21–24.

Add.

21. $(-15) + (-37) + 25 + 42 + (-59) + (-14)$

22. $42 + (-81) + (-28) + 24 + 18 + (-31)$

23. $-2.5 + (-10) + 6 + (-7.5)$

24. $-35 + 17 + 14 + (-27) + 31 + (-12)$

b Opposites, or Additive Inverses

Suppose we add two numbers that are **opposites**, such as 6 and -6. The result is 0. When opposities are added, the result is always 0. Opposites are also called **additive inverses**. Every real number has an opposite, or additive inverse.

OPPOSITES, OR ADDITIVE INVERSES

Two numbers whose sum is 0 are called **opposites**, or **additive inverses**, of each other.

Answers

7. -11 **8.** -12 **9.** 2 **10.** -4
11. -2 **12.** 0 **13.** -22 **14.** 3
15. 0.53 **16.** 2.3 **17.** -7.7 **18.** -6.2
19. $-\frac{2}{9}$ **20.** $-\frac{19}{20}$ **21.** -58 **22.** -56
23. -14 **24.** -12

EXAMPLES Find the opposite, or additive inverse, of each number.

14. 34 The opposite of 34 is -34 because $34 + (-34) = 0$.

15. -8 The opposite of -8 is 8 because $-8 + 8 = 0$.

16. 0 The opposite of 0 is 0 because $0 + 0 = 0$.

17. $-\frac{7}{8}$ The opposite of $-\frac{7}{8}$ is $\frac{7}{8}$ because $-\frac{7}{8} + \frac{7}{8} = 0$.

Find the opposite, or additive inverse, of each number.

25. -4 **26.** 8.7

27. -7.74 **28.** $-\frac{8}{9}$

29. 0 **30.** 12

Do Exercises 25–30.

To name the opposite, we use the symbol $-$, as follows.

SYMBOLIZING OPPOSITES

The opposite, or additive inverse, of a number a can be named $-a$ (read "the opposite of a," or "the additive inverse of a").

Note that if we take a number, say, 8, and find its opposite, -8, and then find the opposite of the result, we will have the original number, 8, again.

THE OPPOSITE OF AN OPPOSITE

The **opposite of the opposite** of a number is the number itself. (The additive inverse of the additive inverse of a number is the number itself.) That is, for any number a,

$$-(-a) = a.$$

EXAMPLE 18 Evaluate $-x$ and $-(-x)$ when $x = 16$.

If $x = 16$, then $-x = -16$. The opposite of 16 is -16.

If $x = 16$, then $-(-x) = -(-16) = 16$. The opposite of the opposite of 16 is 16.

EXAMPLE 19 Evaluate $-x$ and $-(-x)$ when $x = -3$.

If $x = -3$, then $-x = -(-3) = 3$.

If $x = -3$, then $-(-x) = -(-(-3)) = -(3) = -3$.

Note that in Example 19 we used a second set of parentheses to show that we are substituting the negative number -3 for x. Symbolism like $--x$ is not considered meaningful.

Evaluate $-x$ and $-(-x)$ when:

31. $x = 14$. **32.** $x = 1$.

33. $x = -19$. **34.** $x = -1.6$.

35. $x = \frac{2}{3}$. **36.** $x = -\frac{9}{8}$.

Do Exercises 31–36.

A symbol such as -8 is usually read "negative 8." It could be read "the additive inverse of 8," because the additive inverse of 8 is negative 8. It could also be read "the opposite of 8," because the opposite of 8 is -8. Thus a symbol like -8 can be read in more than one way. It is never correct to read -8 as "minus 8."

Caution!

A symbol like $-x$, which has a variable, should be read "the opposite of x" or "the additive inverse of x" and *not* "negative x," because we do not know whether x represents a positive number, a negative number, or 0. You can check this in Examples 18 and 19.

Answers

25. 4 **26.** -8.7 **27.** 7.74 **28.** $\frac{8}{9}$

29. 0 **30.** -12 **31.** -14; 14

32. -1; 1 **33.** 19; -19 **34.** 1.6; -1.6

35. $-\frac{2}{3}$; $\frac{2}{3}$ **36.** $\frac{9}{8}$; $-\frac{9}{8}$

We can use the symbolism $-a$ to restate the definition of opposite, or additive inverse.

OPPOSITES, OR ADDITIVE INVERSES

For any real number a, the **opposite**, or **additive inverse**, of a, denoted $-a$, is such that

$$a + (-a) = (-a) + a = 0.$$

Signs of Numbers

A negative number is sometimes said to have a "negative sign." A positive number is said to have a "positive sign." When we replace a number with its opposite, we can say that we have "changed its sign."

EXAMPLES Find the opposite. (Change the sign.)

20. -3 $\quad -(-3) = 3$

21. $-\frac{2}{13}$ $\quad -\left(-\frac{2}{13}\right) = \frac{2}{13}$

22. 0 $\quad -(0) = 0$

23. 14 $\quad -(14) = -14$

Do Exercises 37–40.

Find the opposite. (Change the sign.)

37. -4

38. -13.4

39. 0

40. $\frac{1}{4}$

C Applications and Problem Solving

Addition of real numbers occurs in many real-world situations.

EXAMPLE 24 *Lake Level.* In the course of one four-month period, the water level of Lake Clearwater went down 2 ft, up 1 ft, down 5 ft, and up 3 ft. By how much had the lake level changed at the end of the four months?

We let $T =$ the total change in the level of the lake. Then the problem translates to a sum:

$$\begin{array}{ccccccccc} \underbrace{\text{Total change}} & \text{is} & \underbrace{\text{1st change}} & \text{plus} & \underbrace{\text{2nd change}} & \text{plus} & \underbrace{\text{3rd change}} & \text{plus} & \underbrace{\text{4th change}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ T & = & -2 & + & 1 & + & (-5) & + & 3. \end{array}$$

Adding from left to right, we have

$$\begin{aligned} T = -2 + 1 + (-5) + 3 &= -1 + (-5) + 3 \\ &= -6 + 3 \\ &= -3. \end{aligned}$$

The lake level had dropped 3 ft at the end of the four-month period.

Do Exercise 41.

41. Change in Class Size. During the first two weeks of the semester in Jim's algebra class, 4 students withdrew, 8 students enrolled late, and 6 students were dropped as "no shows." By how many students had the class size changed at the end of the first two weeks?

Answers

37. 4 **38.** 13.4 **39.** 0

40. $-\frac{1}{4}$ **41.** -2 students

1.3 Exercise Set

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READ

REVIEW

a Add. Do not use the number line except as a check.

1. $2 + (-9)$

2. $-5 + 2$

3. $-11 + 5$

4. $4 + (-3)$

5. $-6 + 6$

6. $8 + (-8)$

7. $-3 + (-5)$

8. $-4 + (-6)$

9. $-7 + 0$

10. $-13 + 0$

11. $0 + (-27)$

12. $0 + (-35)$

13. $17 + (-17)$

14. $-15 + 15$

15. $-17 + (-25)$

16. $-24 + (-17)$

17. $18 + (-18)$

18. $-13 + 13$

19. $-28 + 28$

20. $11 + (-11)$

21. $8 + (-5)$

22. $-7 + 8$

23. $-4 + (-5)$

24. $10 + (-12)$

25. $13 + (-6)$

26. $-3 + 14$

27. $-25 + 25$

28. $50 + (-50)$

29. $53 + (-18)$

30. $75 + (-45)$

31. $-8.5 + 4.7$

32. $-4.6 + 1.9$

33. $-2.8 + (-5.3)$

34. $-7.9 + (-6.5)$

35. $-\frac{3}{5} + \frac{2}{5}$

36. $-\frac{4}{3} + \frac{2}{3}$

37. $-\frac{2}{9} + \left(-\frac{5}{9}\right)$

38. $-\frac{4}{7} + \left(-\frac{6}{7}\right)$

39. $-\frac{5}{8} + \frac{1}{4}$

40. $-\frac{5}{6} + \frac{2}{3}$

41. $-\frac{5}{8} + \left(-\frac{1}{6}\right)$

42. $-\frac{5}{6} + \left(-\frac{2}{9}\right)$

43. $-\frac{3}{8} + \frac{5}{12}$

44. $-\frac{7}{16} + \frac{7}{8}$

45. $-\frac{1}{6} + \frac{7}{10}$

46. $-\frac{11}{18} + \left(-\frac{3}{4}\right)$

47. $\frac{7}{15} + \left(-\frac{1}{9}\right)$

48. $-\frac{4}{21} + \frac{3}{14}$

49. $76 + (-15) + (-18) + (-6)$

50. $29 + (-45) + 18 + 32 + (-96)$

51. $-44 + \left(-\frac{3}{8}\right) + 95 + \left(-\frac{5}{8}\right)$

52. $24 + 3.1 + (-44) + (-8.2) + 63$

53. $98 + (-54) + 113 + (-998) + 44 + (-612)$

54. $-458 + (-124) + 1025 + (-917) + 218$

b Find the opposite, or additive inverse.

55. 24

56. -64

57. -26.9

58. 48.2

Evaluate $-x$ when:

59. $x = 8$.

60. $x = -27$.

61. $x = -\frac{13}{8}$.

62. $x = \frac{1}{236}$.

Evaluate $-(-x)$ when:

63. $x = -43$.

64. $x = 39$.

65. $x = \frac{4}{3}$.

66. $x = -7.1$.

Find the opposite. (Change the sign.)

67. -24

68. -12.3

69. $-\frac{3}{8}$

70. 10

c Solve.

71. ***Tallest Mountain.*** The tallest mountain in the world, when measured from base to peak, is Mauna Kea (White Mountain) in Hawaii. From its base 19,684 ft below sea level in the Hawaiian Trough, it rises 33,480 ft. What is the elevation of the peak above sea level?

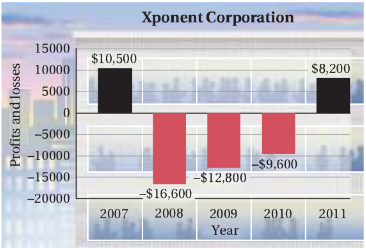
Source: *The Guinness Book of Records*

72. ***Telephone Bills.*** Erika's cell-phone bill for July was $82. She sent a check for $50 and then made $37 worth of calls in August. How much did she then owe on her cell-phone bill?

73. ***Temperature Changes.*** One day the temperature in Lawrence, Kansas, is 32°F at 6:00 A.M. It rises 15° by noon, but falls 50° by midnight when a cold front moves in. What is the final temperature?

74. ***Stock Changes.*** On a recent day, the price of a stock opened at a value of $61.38. During the day, it rose $4.75, dropped $7.38, and rose $5.13. Find the value of the stock at the end of the day.

75. *Profits and Losses.* The profit of a business is expressed as a positive number and referred to as operating "in the black." A loss is expressed as a negative number and is referred to as operating "in the red." The profits and losses of Xponent Corporation over various years are shown in the bar graph below. Find the sum of the profits and losses.



76. *Football Yardage.* In a college football game, the quarterback attempted passes with the following results. Find the total gain or loss.

TRY	GAIN OR LOSS
1st	13-yd gain
2nd	12-yd loss
3rd	21-yd gain

77. *Credit-Card Bills.* On August 1, Lyle's credit-card bill shows that he owes $470. During the month of August, Lyle sends a check for $45 to the credit-card company, charges another $160 in merchandise, and then pays off another $500 of his bill. What is the new amount that Lyle owes at the end of August?

78. *Account Balance.* Leah has $460 in a checking account. She writes a check for $530, makes a deposit of $75, and then writes a check for $90. What is the balance in her account?

Skill Maintenance

Convert to decimal notation. [1.2c]

79. $-\frac{5}{8}$

80. $\frac{1}{3}$

81. $-\frac{1}{12}$

82. $\frac{13}{20}$

Find the absolute value. [1.2e]

83. $|2.3|$

84. $|0|$

85. $\left|-\frac{4}{5}\right|$

86. $|-21.4|$

Synthesis

87. For what numbers x is $-x$ negative?

88. For what numbers x is $-x$ positive?

89. If a is positive and b is negative, then $-a + b$ is:

A. Positive.
B. Negative.
C. 0.
D. Cannot be determined without more information

90. If $a = b$ and a and b are negative, then $-a + (-b)$ is:

A. Positive.
B. Negative.
C. 0.
D. Cannot be determined without more information

1.4 Subtraction of Real Numbers

OBJECTIVES

a Subtract real numbers and simplify combinations of additions and subtractions.

b Solve applied problems involving subtraction of real numbers.

a Subtraction

We now consider subtraction of real numbers.

SUBTRACTION

The difference $a - b$ is the number c for which $a = b + c$.

Consider, for example, $45 - 17$. *Think*: What number can we add to 17 to get 45? Since $45 = 17 + 28$, we know that $45 - 17 = 28$. Let's consider an example whose answer is a negative number.

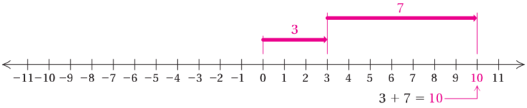
EXAMPLE 1 Subtract: $3 - 7$.

Think: What number can we add to 7 to get 3? The number must be negative. Since $7 + (-4) = 3$, we know the number is -4: $3 - 7 = -4$. That is, $3 - 7 = -4$ because $7 + (-4) = 3$.

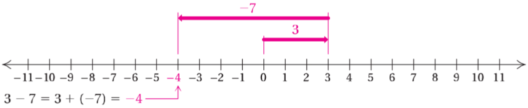
Do Exercises 1–3.

The definition above does not provide the most efficient way to do subtraction. We can develop a faster way to subtract. As a rationale for the faster way, let's compare $3 + 7$ and $3 - 7$ on the number line.

To find $3 + 7$ on the number line, we start at 0, move to 3, and then move 7 units farther to the right since 7 is positive.



To find $3 - 7$, we do the "opposite" of adding 7: We move 7 units to the *left* to do the subtracting. This is the same as *adding* the opposite of 7, -7, to 3.



Do Exercises 4–6.

Look for a pattern in the examples shown at right.

SUBTRACTING	ADDING AN OPPOSITE
$5 - 8 = -3$	$5 + (-8) = -3$
$-6 - 4 = -10$	$-6 + (-4) = -10$
$-7 - (-2) = -5$	$-7 + 2 = -5$

Subtract.

1. $-6 - 4$

Think: What number can be added to 4 to get -6:

☐ $+ 4 = -6$?

2. $-7 - (-10)$

Think: What number can be added to -10 to get -7:

☐ $+ (-10) = -7$?

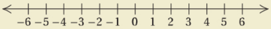
3. $-7 - (-2)$

Think: What number can be added to -2 to get -7:

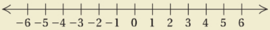
☐ $+ (-2) = -7$?

Subtract. Use the number line, doing the "opposite" of addition.

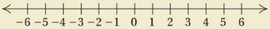
4. $5 - 9$



5. $-3 - 2$



6. $-4 - (-3)$



Answers

1. -10 **2.** 3 **3.** -5 **4.** -4
5. -5 **6.** -1

Complete the addition and compare with the subtraction.

7. $4 - 6 = -2;$
$4 + (-6) =$ ________

8. $-3 - 8 = -11;$
$-3 + (-8) =$ ________

9. $-5 - (-9) = 4;$
$-5 + 9 =$ ________

10. $-5 - (-3) = -2;$
$-5 + 3 =$ ________

Do Exercises 7–10.

Perhaps you have noticed that we can subtract by adding the opposite of the number being subtracted. This can always be done.

SUBTRACTING BY ADDING THE OPPOSITE

For any real numbers a and b,

$$a - b = a + (-b).$$

(To subtract, add the opposite, or additive inverse, of the number being subtracted.)

This is the method generally used for quick subtraction of real numbers.

EXAMPLES Subtract.

2. $2 - 6 = 2 + (-6) = -4$

The opposite of 6 is -6. We change the subtraction to addition and add the opposite.
Check: $-4 + 6 = 2.$

3. $4 - (-9) = 4 + 9 = 13$

The opposite of -9 is 9. We change the subtraction to addition and add the opposite.
Check: $13 + (-9) = 4.$

4. $-4.2 - (-3.6) = -4.2 + 3.6 = -0.6$

Adding the opposite.
Check: $-0.6 + (-3.6) = -4.2.$

5. $$-\frac{1}{2} - \left(-\frac{3}{4}\right) = -\frac{1}{2} + \frac{3}{4} = -\frac{2}{4} + \frac{3}{4} = \frac{1}{4}$$

Adding the opposite.
Check: $\frac{1}{4} + \left(-\frac{3}{4}\right) = -\frac{1}{2}.$

Subtract.

11. $2 - 8$

12. $-6 - 10$

13. $12.4 - 5.3$

14. $-8 - (-11)$

15. $-8 - (-8)$

16. $\frac{2}{3} - \left(-\frac{5}{6}\right)$

Do Exercises 11–16.

EXAMPLES Subtract by adding the opposite of the number being subtracted.

6. $3 - 5$ *Think*: "Three minus five is three plus the opposite of five"

$3 - 5 = 3 + (-5) = -2$

7. $\frac{1}{8} - \frac{7}{8}$ *Think*: "One-eighth minus seven-eighths is one-eighth plus the opposite of seven-eighths"

$$\frac{1}{8} - \frac{7}{8} = \frac{1}{8} + \left(-\frac{7}{8}\right) = -\frac{6}{8}, \text{ or } -\frac{3}{4}$$

8. $-4.6 - (-9.8)$ *Think*: "Negative four point six minus negative nine point eight is negative four point six plus the opposite of negative nine point eight"

$-4.6 - (-9.8) = -4.6 + 9.8 = 5.2$

9. $-\frac{3}{4} - \frac{7}{5}$ *Think*: "Negative three-fourths minus seven-fifths is negative three-fourths plus the opposite of seven-fifths"

$$-\frac{3}{4} - \frac{7}{5} = -\frac{3}{4} + \left(-\frac{7}{5}\right) = -\frac{15}{20} + \left(-\frac{28}{20}\right) = -\frac{43}{20}$$

Subtract by adding the opposite of the number being subtracted.

17. $3 - 11$

18. $12 - 5$

19. $-12 - (-9)$

20. $-12.4 - 10.9$

21. $-\frac{4}{5} - \left(-\frac{4}{5}\right)$

Do Exercises 17–21.

Answers

7. -2 8. -11 9. 4 10. -2 11. -6
12. -16 13. 7.1 14. 3 15. 0 16. $\frac{3}{2}$
17. -8 18. 7 19. -3 20. -23.3
21. 0

When several additions and subtractions occur together, we can make them all additions.

EXAMPLES Simplify.

10. $8 - (-4) - 2 - (-4) + 2 = 8 + 4 + (-2) + 4 + 2$ Adding the opposite

$= 16$

11. $8.2 - (-6.1) + 2.3 - (-4) = 8.2 + 6.1 + 2.3 + 4 = 20.6$

12.
$$\frac{3}{4} - \left(-\frac{1}{12}\right) - \frac{5}{6} - \frac{2}{3} = \frac{9}{12} + \frac{1}{12} + \left(-\frac{10}{12}\right) + \left(-\frac{8}{12}\right)$$
$$= \frac{9 + 1 + (-10) + (-8)}{12}$$
$$= \frac{-8}{12} = -\frac{8}{12} = -\frac{2}{3}$$

Do Exercises 22–24.

b Applications and Problem Solving

Let's now see how we can use subtraction of real numbers to solve applied problems.

EXAMPLE 13 *Surface Temperatures on Mars.* Surface temperatures on Mars vary from −128°C during polar night to 27°C at the equator during midday at the closest point in orbit to the sun. Find the difference between the highest value and the lowest value in this temperature range.

Source: Mars Institute

We let $D =$ the difference in the temperatures. Then the problem translates to the following subtraction:

$$\begin{array}{ccccc} \underbrace{\text{Difference in temperature}} & \text{is} & \underbrace{\text{Highest temperature}} & \text{minus} & \underbrace{\text{Lowest temperature}} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ D & = & 27 & - & (-128) \\ D & = & 27 + 128 = 155. & & \end{array}$$

The difference in the temperatures is 155°C.

Do Exercise 25.

Simplify.

22. $-6 - (-2) - (-4) - 12 + 3$

23. $\frac{2}{3} - \frac{4}{5} - \left(-\frac{11}{15}\right) + \frac{7}{10} - \frac{5}{2}$

24. $-9.6 + 7.4 - (-3.9) - (-11)$

25. Temperature Extremes. The highest temperature ever recorded in the United States is 134°F in Greenland Ranch, California, on July 10, 1913. The lowest temperature ever recorded is −80°F in Prospect Creek, Alaska, on January 23, 1971. How much higher was the temperature in Greenland Ranch than the temperature in Prospect Creek?

Source: National Oceanographic and Atmospheric Administration

Answers

22. -9 **23.** $-\frac{6}{5}$ **24.** 12.7 **25.** 214°F

1.4 Exercise Set

For Extra Help

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a Subtract.

1. $2 - 9$

2. $3 - 8$

3. $-8 - (-2)$

4. $-6 - (-8)$

5. $-11 - (-11)$

6. $-6 - (-6)$

7. $12 - 16$

8. $14 - 19$

9. $20 - 27$

10. $30 - 4$

11. $-9 - (-3)$

12. $-7 - (-9)$

13. $-40 - (-40)$

14. $-9 - (-9)$

15. $7 - (-7)$

16. $4 - (-4)$

17. $8 - (-3)$

18. $-7 - 4$

19. $-6 - 8$

20. $6 - (-10)$

21. $-4 - (-9)$

22. $-14 - 2$

23. $-6 - (-5)$

24. $-4 - (-3)$

25. $8 - (-10)$

26. $5 - (-6)$

27. $-5 - (-2)$

28. $-3 - (-1)$

29. $-7 - 14$

30. $-9 - 16$

31. $0 - (-5)$

32. $0 - (-1)$

33. $-8 - 0$

34. $-9 - 0$

35. $7 - (-5)$

36. $7 - (-4)$

37. $2 - 25$

38. $18 - 63$

39. $-42 - 26$

40. $-18 - 63$

41. $-71 - 2$

42. $-49 - 3$

43. $24 - (-92)$

44. $48 - (-73)$

45. $-50 - (-50)$

46. $-70 - (-70)$

47. $-\frac{3}{8} - \frac{5}{8}$

48. $\frac{3}{9} - \frac{9}{9}$

49. $\frac{3}{4} - \frac{2}{3}$

50. $\frac{5}{8} - \frac{3}{4}$

51. $-\frac{3}{4} - \frac{2}{3}$

52. $-\frac{5}{8} - \frac{3}{4}$

53. $-\frac{5}{8} - \left(-\frac{3}{4}\right)$

54. $-\frac{3}{4} - \left(-\frac{2}{3}\right)$

55. $6.1 - (-13.8)$

56. $1.5 - (-3.5)$

57. $-2.7 - 5.9$

58. $-3.2 - 5.8$

59. $0.99 - 1$

60. $0.87 - 1$

61. $-79 - 114$

62. $-197 - 216$

63. $0 - (-500)$

64. $500 - (-1000)$

65. $-2.8 - 0$

66. $6.04 - 1.1$

67. $7 - 10.53$

68. $8 - (-9.3)$

69. $\frac{1}{6} - \frac{2}{3}$

70. $-\frac{3}{8} - \left(-\frac{1}{2}\right)$

71. $-\frac{4}{7} - \left(-\frac{10}{7}\right)$

72. $\frac{12}{5} - \frac{12}{5}$

73. $-\frac{7}{10} - \frac{10}{15}$

74. $-\frac{4}{18} - \left(-\frac{2}{9}\right)$

75. $\frac{1}{5} - \frac{1}{3}$

76. $-\frac{1}{7} - \left(-\frac{1}{6}\right)$

77. $\frac{5}{12} - \frac{7}{16}$

78. $-\frac{1}{35} - \left(-\frac{9}{40}\right)$

79. $-\frac{2}{15} - \frac{7}{12}$

80. $\frac{2}{21} - \frac{9}{14}$

Simplify.

81. $18 - (-15) - 3 - (-5) + 2$

82. $22 - (-18) + 7 + (-42) - 27$

83. $-31 + (-28) - (-14) - 17$

84. $-43 - (-19) - (-21) + 25$

85. $-34 - 28 + (-33) - 44$

86. $39 + (-88) - 29 - (-83)$

87. $-93 - (-84) - 41 - (-56)$

88. $84 + (-99) + 44 - (-18) - 43$

89. $-5.4 - (-30.9) + 30.8 + 40.2 - (-12)$

90. $14.9 - (-50.7) + 20 - (-32.8)$

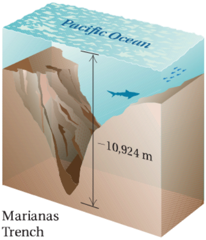
91. $-\frac{7}{12} + \frac{3}{4} - \left(-\frac{5}{8}\right) - \frac{13}{24}$

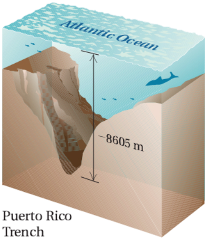
92. $-\frac{11}{16} + \frac{5}{32} - \left(-\frac{1}{4}\right) + \frac{7}{8}$

b Solve.

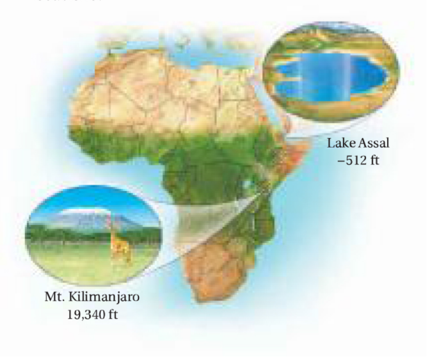
93. ***Ocean Depth.*** The deepest point in the Pacific Ocean is the Marianas Trench, with a depth of 10,924 m. The deepest point in the Atlantic Ocean is the Puerto Rico Trench, with a depth of 8605 m. What is the difference in the elevation of the two trenches?

Source: *The World Almanac and Book of Facts*





94. ***Elevations in Africa.*** The elevation of the highest point in Africa, Mt. Kilimanjaro, Tanzania, is 19,340 ft. The lowest elevation, at Lake Assal, Djibouti, is −512 ft. What is the difference in the elevations of the two locations?



95. Claire has a charge of $476.89 on her credit card, but she then returns a sweater that cost $128.95. How much does she now owe on her credit card?

96. Chris has $720 in a checking account. He writes a check for $970 to pay for a sound system. What is the balance in his checking account?

97. ***Difference in Elevation.*** At its highest point, the elevation of Denver, Colorado, is 5672 ft above sea level. At its lowest point, the elevation of New Orleans, Louisiana, is 4 ft below sea level. Find the difference in the elevations.

Source: *Information Please Almanac*

99. ***Low Points on Continents.*** The lowest point in Africa is Lake Assal, which is 512 ft below sea level. The lowest point in South America is the Valdes Peninsula, which is 131 ft below sea level. How much lower is Lake Assal than the Valdes Peninsula?

Source: National Geographic Society

101. ***Surface Temperature on Mercury.*** Surface temperatures on Mercury vary from 840°F on the equator when the planet is closest to the sun to -290°F at night. Find the difference between these two temperatures.

98. ***Difference in Elevation.*** The lowest elevation in North America, Death Valley, California, is 282 ft below sea level. The highest elevation in North America, Mount McKinley, Alaska, is 20,320 ft. Find the difference in elevation between the highest point and the lowest point.

Source: National Geographic Society

100. ***Temperature Records.*** The greatest recorded temperature change in one 24-hr period occurred between January 23 and January 24, 1916, in Browning, Montana, where the temperature fell from to 44°F to -56°F. By how much did the temperature drop?

Source: *The Guinness Book of Records*

102. ***Run Differential.*** In baseball, the difference between the number of runs that a team scores and the number of runs that it allows its opponents to score is called the *run differential.* That is,

$$\text{Run differential} = \begin{matrix}\text{Number of}\\ \text{runs scored}\end{matrix} - \begin{matrix}\text{Number of}\\ \text{runs allowed.}\end{matrix}$$

Teams strive for a positive run differential.

Source: Major League Baseball

a) In a recent season, the Chicago White Sox scored 810 runs and allowed 729 runs to be scored on them. Find the run differential.

b) In a recent season, the Pittsburgh Pirates scored 735 runs and allowed 884 runs to be scored on them. Find the run differential.

Skill Maintenance

Translate to an algebraic expression. [1.1b]

103. 7 more than y

104. 41 less than t

105. h subtracted from a

106. The product of 6 and c

107. r more than s

108. x less than y

Synthesis

Determine whether each statement is true or false for all integers a and b. If false, give an example to show why. Examples may vary.

109. $a - 0 = 0 - a$

110. $0 - a = a$

111. If $a \neq b$, then $a - b \neq 0$.

112. If $a = -b$, then $a + b = 0$.

113. If $a + b = 0$, then a and b are opposites.

114. If $a - b = 0$, then $a = -b$.

Mid-Chapter Review

Concept Reinforcement

Determine whether each statement is true or false.

__________ **1.** All rational numbers can be named using fraction notation. [1.2c]

__________ **2.** If $a > b$, then a lies to the left of b on the number line. [1.2d]

__________ **3.** The absolute value of a number is always nonnegative. [1.2e]

__________ **4.** We can translate "7 less than y" as $7 - y$. [1.1b]

Guided Solutions

Fill in each blank with the number that creates a correct statement or solution.

5. Evaluate $-x$ and $-(-x)$ when $x = -4$. [1.3b]

$-x = -(\square) = \square$;

$-(-x) = -(-(\square)) = -(\square) = \square$

Subtract. [1.4a]

6. $5 - 13 = 5 + (\square) = \square$

7. $-6 - 7 = -6 + (\square) = \square$

Mixed Review

Evaluate. [1.1a]

8. $\frac{3m}{n}$, when $m = 8$ and $n = 6$

9. $\frac{a + b}{2}$, when $a = 5$ and $b = 17$

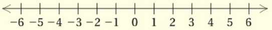
Translate each phrase to an algebraic expression. Use any letter for the variable. [1.1b]

10. Three times some number

11. Five less than some number

12. State the integers that correspond to this situation: Jerilyn deposited \$450 in her checking account. Later that week, she wrote a check for \$79. [1.2a]

13. Graph -3.5 on the number line. [1.2b]



Convert to decimal notation. [1.2c]

14. $-\frac{4}{5}$

15. $\frac{7}{3}$

Use either $<$ or $>$ for $\square$ to write a true sentence. [1.2d]

16. $-5 \square -3$

17. $-9.9 \square -10.1$

Write true or false. [1.2d]

18. $-8 \geq -5$

19. $-4 \leq -4$

Write an inequality with the same meaning. [1.2d]

20. $y < 5$

21. $-3 \geq t$

Find the absolute value. [1.2e]

22. $|15.6|$

23. $|-18|$

24. $|0|$

25. $\left|-\frac{12}{5}\right|$

Find the opposite, or additive inverse, of the number. [1.3b]

26. -5.6

27. $\frac{7}{4}$

28. 0

29. -49

30. Evaluate $-x$ when x is -19. [1.3b]

31. Evaluate $-(-x)$ when x is 2.3. [1.3b]

Compute and simplify. [1.3a], [1.4a]

32. $7 + (-9)$

33. $-\frac{3}{8} + \frac{1}{4}$

34. $3.6 + (-3.6)$

35. $-8 + (-9)$

36. $\frac{2}{3} + \left(-\frac{9}{8}\right)$

37. $-4.2 + (-3.9)$

38. $-14 + 5$

39. $19 + (-21)$

40. $-4.1 - 6.3$

41. $5 - (-11)$

42. $-\frac{1}{4} - \left(-\frac{3}{5}\right)$

43. $12 - 24$

44. $-8 - (-4)$

45. $-\frac{1}{2} - \frac{5}{6}$

46. $12.3 - 14.1$

47. $6 - (-7)$

48. $16 - (-9) - 20 - (-4)$

49. $-4 + (-10) - (-3) - 12$

50. $17 - (-25) + 15 - (-18)$

51. $-9 + (-3) + 16 - (-10)$

Solve. [1.3c], [1.4b]

52. *Temperature Change.* In chemistry lab, Ben works with a substance whose initial temperature is 25°C. During an experiment, the temperature falls to −8°C. Find the difference between the two temperatures.

53. *Stock Price Change.* The price of a stock opened at \$56.12. During the day, it dropped \$1.18, then rose \$1.22, and then dropped \$1.36. Find the value of the stock at the end of the day.

Understanding Through Discussion and Writing

54. Give three examples of rational numbers that are not integers. Explain. [1.2b]

56. Explain in your own words why the sum of two negative numbers is always negative. [1.3a]

55. Give three examples of irrational numbers. Explain the difference between an irrational number and a rational number. [1.2b, d]

57. If a negative number is subtracted from a positive number, will the result always be positive? Why or why not? [1.4a]

1.5

Multiplication of Real Numbers

OBJECTIVES

- **a** Multiply real numbers.
- **b** Solve applied problems involving multiplication of real numbers.

a Multiplication

Multiplication of real numbers is very much like multiplication of arithmetic numbers. The only difference is that we must determine whether the answer is positive or negative.

Multiplication of a Positive Number and a Negative Number

To see how to multiply a positive number and a negative number, consider the pattern of the following.

This number decreases by 1 each time.

$4 \cdot 5 = 20$
$3 \cdot 5 = 15$
$2 \cdot 5 = 10$
$1 \cdot 5 = 5$
$0 \cdot 5 = 0$
$-1 \cdot 5 = -5$
$-2 \cdot 5 = -10$
$-3 \cdot 5 = -15$

This number decreases by 5 each time.

Do Exercise 1.

According to this pattern, it looks as though the product of a negative number and a positive number is negative. That is the case, and we have the first part of the rule for multiplying real numbers.

THE PRODUCT OF A POSITIVE NUMBER AND A NEGATIVE NUMBER

To multiply a positive number and a negative number, multiply their absolute values. The answer is negative.

EXAMPLES Multiply.

1. $8(-5) = -40$

2. $-\frac{1}{3} \cdot \frac{5}{7} = -\frac{5}{21}$

3. $(-7.2)5 = -36$

Do Exercises 2–7.

1. Complete, as in the example.

$4 \cdot 10 = 40$
$3 \cdot 10 = 30$
$2 \cdot 10 =$
$1 \cdot 10 =$
$0 \cdot 10 =$
$-1 \cdot 10 =$
$-2 \cdot 10 =$
$-3 \cdot 10 =$

Multiply.

2. $-3 \cdot 6$

3. $20 \cdot (-5)$

4. $4 \cdot (-20)$

5. $-\frac{2}{3} \cdot \frac{5}{6}$

6. $-4.23(7.1)$

7. $\frac{7}{8}\left(-\frac{4}{5}\right)$

Answers

1. 20; 10; 0; −10; −20; −30 **2.** −18
3. −100 **4.** −80 **5.** $-\frac{5}{9}$
6. −30.033 **7.** $-\frac{7}{10}$

Multiplication of Two Negative Numbers

How do we multiply two negative numbers? Again, we look for a pattern.

This number decreases by 1 each time.

$$\begin{aligned} 4\cdot(-5) &= -20 \\ 3\cdot(-5) &= -15 \\ 2\cdot(-5) &= -10 \\ 1\cdot(-5) &= -5 \\ 0\cdot(-5) &= 0 \\ -1\cdot(-5) &= 5 \\ -2\cdot(-5) &= 10 \\ -3\cdot(-5) &= 15 \end{aligned}$$

This number increases by 5 each time.

Do Exercise 8.

According to the pattern, it appears that the product of two negative numbers is positive. That is actually so, and we have the second part of the rule for multiplying real numbers.

THE PRODUCT OF TWO NEGATIVE NUMBERS

To multiply two negative numbers, multiply their absolute values. The answer is positive.

Do Exercises 9–14.

The following is another way to consider the rules we have for multiplication.

To multiply two nonzero real numbers:

a) Multiply the absolute values.
b) If the signs are the same, the answer is positive.
c) If the signs are different, the answer is negative.

Multiplication by Zero

The only case that we have not considered is multiplying by zero. As with nonnegative numbers, the product of any real number and 0 is 0.

THE MULTIPLICATION PROPERTY OF ZERO

For any real number a,

$$a\cdot 0 = 0\cdot a = 0.$$

(The product of 0 and any real number is 0.)

EXAMPLES Multiply.

4. $(-3)(-4) = 12$

5. $-1.6(2) = -3.2$

6. $-19\cdot 0 = 0$

7. $\left(-\frac{5}{6}\right)\left(-\frac{1}{9}\right) = \frac{5}{54}$

8. $0\cdot(-452) = 0$

9. $23\cdot 0\cdot\left(-8\frac{2}{3}\right) = 0$

Do Exercises 15–20.

8. Complete, as in the example.

$$\begin{aligned} 3\cdot(-10) &= -30 \\ 2\cdot(-10) &= -20 \\ 1\cdot(-10) &= \\ 0\cdot(-10) &= \\ -1\cdot(-10) &= \\ -2\cdot(-10) &= \\ -3\cdot(-10) &= \end{aligned}$$

Multiply.

9. $-9\cdot(-3)$

10. $-16\cdot(-2)$

11. $-7\cdot(-5)$

12. $-\frac{4}{7}\left(-\frac{5}{9}\right)$

13. $-\frac{3}{2}\left(-\frac{4}{9}\right)$

14. $-3.25(-4.14)$

Multiply.

15. $5(-6)$

16. $(-5)(-6)$

17. $(-3.2)\cdot 10$

18. $\left(-\frac{4}{5}\right)\left(\frac{10}{3}\right)$

19. $0\cdot(-34.2)$

20. $-\frac{5}{7}\cdot 0\cdot\left(-4\frac{2}{3}\right)$

Answers

8. −10; 0; 10; 20; 30 **9.** 27 **10.** 32
11. 35 **12.** $\frac{20}{63}$ **13.** $\frac{2}{3}$ **14.** 13.455
15. −30 **16.** 30 **17.** −32 **18.** $-\frac{8}{3}$
19. 0 **20.** 0

Multiplying More Than Two Numbers

When multiplying more than two real numbers, we can choose order and grouping as we please.

EXAMPLES Multiply.

10. $-8 \cdot 2(-3) = -16(-3)$ Multiplying the first two numbers
$= 48$

11. $-8 \cdot 2(-3) = 24 \cdot 2$ Multiplying the negatives. Every pair of negative numbers gives a positive product.
$= 48$

12. $-3(-2)(-5)(4) = 6(-5)(4)$ Multiplying the first two numbers
$= (-30)4$
$= -120$

13. $\left(-\frac{1}{2}\right)(8)\left(-\frac{2}{3}\right)(-6) = (-4)4$ Multiplying the first two numbers and the last two numbers
$= -16$

14. $-5 \cdot (-2) \cdot (-3) \cdot (-6) = 10 \cdot 18 = 180$

15. $(-3)(-5)(-2)(-3)(-6) = (-30)(18) = -540$

Considering that the product of a pair of negative numbers is positive, we see the following pattern.

The product of an even number of negative numbers is positive.
The product of an odd number of negative numbers is negative.

Do Exercises 21–26.

EXAMPLE 16 Evaluate $2x^2$ when $x = 3$ and when $x = -3$.

$2x^2 = 2(3)^2 = 2(9) = 18;$
$2x^2 = 2(-3)^2 = 2(9) = 18$

Let's compare the expressions $(-x)^2$ and $-x^2$.

EXAMPLE 17 Evaluate $(-x)^2$ and $-x^2$ when $x = 5$.

$(-x)^2 = (-5)^2 = (-5)(-5) = 25;$ Substitute 5 for x. Then evaluate the power.

$-x^2 = -(5)^2 = -(25) = -25$ Substitute 5 for x. Evaluate the power. Then find the opposite.

In Example 17, we see that the expressions $(-x)^2$ and $-x^2$ are *not* equivalent. That is, they do not have the same value for every allowable replacement of the variable by a real number. To find $(-x)^2$, we take the opposite and then square. To find $-x^2$, we find the square and then take the opposite.

Multiply.

21. $5 \cdot (-3) \cdot 2$

22. $-3 \times (-4.1) \times (-2.5)$

23. $-\frac{1}{2} \cdot \left(-\frac{4}{3}\right) \cdot \left(-\frac{5}{2}\right)$

24. $-2 \cdot (-5) \cdot (-4) \cdot (-3)$

25. $(-4)(-5)(-2)(-3)(-1)$

26. $(-1)(-1)(-2)(-3)(-1)(-1)$

Answers

21. -30 **22.** -30.75 **23.** $-\frac{5}{3}$
24. 120 **25.** -120 **26.** 6

EXAMPLE 18 Evaluate $(-a)^2$ and $-a^2$ when $a = -4$.

To make sense of the substitutions and computations, we introduce extra grouping symbols into the expressions.

$$(-a)^2 = [-(-4)]^2 = [4]^2 = 16;$$
$$-a^2 = -(-4)^2 = -(16) = -16$$

Do Exercises 27–29.

27. Evaluate $3x^2$ when $x = 4$ and when $x = -4$.

28. Evaluate $(-x)^2$ and $-x^2$ when $x = 2$.

29. Evaluate $(-x)^2$ and $-x^2$ when $x = -3$.

b Applications and Problem Solving

We now consider multiplication of real numbers in real-world applications.

EXAMPLE 19 *Chemical Reaction.* During a chemical reaction, the temperature in a beaker decreased by 2°C every minute until 10:23 A.M. If the temperature was 17°C at 10:00 A.M., when the reaction began, what was the temperature at 10:23 A.M.?

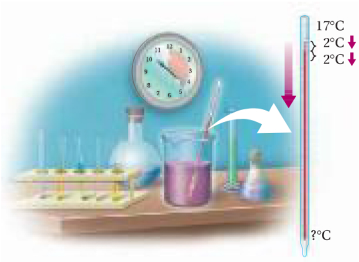
This is a multistep problem. We first find the total number of degrees that the temperature dropped, using −2° for each minute. Since it dropped 2° for each of the 23 minutes, we know that the total drop d is given by

$$d = 23 \cdot (-2) = -46.$$

To determine the temperature after this time period, we find the sum of 17 and −46, or

$$T = 17 + (-46) = -29.$$

Thus the temperature at 10:23 A.M. was −29°C.



Do Exercise 30.

30. Chemical Reaction. During a chemical reaction, the temperature in a beaker increased by 3°C every minute until 1:34 P.M. If the temperature was −17°C at 1:10 P.M., when the reaction began, what was the temperature at 1:34 P.M.?

STUDY TIPS

MAKING POSITIVE CHOICES

Making the right choices can give you the power to succeed in learning mathematics.

You can choose to improve your attitude and raise the academic goals that you have set for yourself. Projecting a positive attitude toward your study of mathematics and expecting a positive outcome can make it easier for you to learn and to perform well in this course.

Here are some positive choices you can make:

- Choose to make a strong commitment to learning.
- Choose to allocate the proper amount of time to learn.
- Choose to place the primary responsibility for learning on yourself.

Well-known American psychologist William James once said, "The one thing that will guarantee the successful conclusion of a doubtful undertaking is faith in the beginning that you can do it."

Answers

27. 48; 48 **28.** 4; −4
29. 9; −9 **30.** 55°C

1.5 Exercise Set

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a Multiply.

1. $-4 \cdot 2$

2. $-3 \cdot 5$

3. $-8 \cdot 6$

4. $-5 \cdot 2$

5. $8 \cdot (-3)$

6. $9 \cdot (-5)$

7. $-9 \cdot 8$

8. $-10 \cdot 3$

9. $-8 \cdot (-2)$

10. $-2 \cdot (-5)$

11. $-7 \cdot (-6)$

12. $-9 \cdot (-2)$

13. $15 \cdot (-8)$

14. $-12 \cdot (-10)$

15. $-14 \cdot 17$

16. $-13 \cdot (-15)$

17. $-25 \cdot (-48)$

18. $39 \cdot (-43)$

19. $-3.5 \cdot (-28)$

20. $97 \cdot (-2.1)$

21. $9 \cdot (-8)$

22. $7 \cdot (-9)$

23. $4 \cdot (-3.1)$

24. $3 \cdot (-2.2)$

25. $-5 \cdot (-6)$

26. $-6 \cdot (-4)$

27. $-7 \cdot (-3.1)$

28. $-4 \cdot (-3.2)$

29. $\frac{2}{3} \cdot \left(-\frac{3}{5}\right)$

30. $\frac{5}{7} \cdot \left(-\frac{2}{3}\right)$

31. $-\frac{3}{8} \cdot \left(-\frac{2}{9}\right)$

32. $-\frac{5}{8} \cdot \left(-\frac{2}{5}\right)$

33. -6.3×2.7

34. -4.1×9.5

35. $-\frac{5}{9} \cdot \frac{3}{4}$

36. $-\frac{8}{3} \cdot \frac{9}{4}$

37. $7 \cdot (-4) \cdot (-3) \cdot 5$

38. $9 \cdot (-2) \cdot (-6) \cdot 7$

39. $-\frac{2}{3} \cdot \frac{1}{2} \cdot \left(-\frac{6}{7}\right)$

40. $-\frac{1}{8} \cdot \left(-\frac{1}{4}\right) \cdot \left(-\frac{3}{5}\right)$

41. $-3 \cdot (-4) \cdot (-5)$

42. $-2 \cdot (-5) \cdot (-7)$

43. $-2 \cdot (-5) \cdot (-3) \cdot (-5)$

44. $-3 \cdot (-5) \cdot (-2) \cdot (-1)$

45. $\frac{1}{5}\left(-\frac{2}{9}\right)$

46. $-\frac{3}{5}\left(-\frac{2}{7}\right)$

47. $-7 \cdot (-21) \cdot 13$

48. $-14 \cdot (34) \cdot 12$

49. $-4 \cdot (-1.8) \cdot 7$

50. $-8 \cdot (-1.3) \cdot (-5)$

51. $-\frac{1}{9}\left(-\frac{2}{3}\right)\left(\frac{5}{7}\right)$

52. $-\frac{7}{2}\left(-\frac{5}{7}\right)\left(-\frac{2}{5}\right)$

53. $4 \cdot (-4) \cdot (-5) \cdot (-12)$

54. $-2 \cdot (-3) \cdot (-4) \cdot (-5)$

55. $0.07 \cdot (-7) \cdot 6 \cdot (-6)$

56. $80 \cdot (-0.8) \cdot (-90) \cdot (-0.09)$

57. $\left(-\frac{5}{6}\right)\left(\frac{1}{8}\right)\left(-\frac{3}{7}\right)\left(-\frac{1}{7}\right)$

58. $\left(\frac{4}{5}\right)\left(-\frac{2}{3}\right)\left(-\frac{15}{7}\right)\left(\frac{1}{2}\right)$

59. $(-14) \cdot (-27) \cdot 0$

60. $7 \cdot (-6) \cdot 5 \cdot (-4) \cdot 3 \cdot (-2) \cdot 1 \cdot 0$

61. $(-8)(-9)(-10)$

62. $(-7)(-8)(-9)(-10)$

63. $(-6)(-7)(-8)(-9)(-10)$

64. $(-5)(-6)(-7)(-8)(-9)(-10)$

65. $(-1)^{12}$

66. $(-1)^{9}$

67. Evaluate $(-x)^2$ and $-x^2$ when $x = 4$ and when $x = -4$.

68. Evaluate $(-x)^2$ and $-x^2$ when $x = 10$ and when $x = -10$.

69. Evaluate $(-3x)^2$ and $-3x^2$ when $x = 7$.

70. Evaluate $(-2x)^2$ and $-2x^2$ when $x = 3$.

71. Evaluate $5x^2$ when $x = 2$ and when $x = -2$.

72. Evaluate $2x^2$ when $x = 5$ and when $x = -5$.

73. Evaluate $-2x^3$ when $x = 1$ and when $x = -1$.

74. Evaluate $-3x^3$ when $x = 2$ and when $x = -2$.

b Solve.

75. ***Weight Loss.*** Dave lost 2 lb each week for a period of 10 weeks. Express his total weight change as an integer.

76. ***Stock Loss.*** Emma lost \$3 each day for a period of 5 days in the value of a stock she owned. Express her total loss as an integer.

77. ***Chemical Reaction.*** The temperature of a chemical compound was 0°C at 11:00 A.M. During a reaction, it dropped 3°C per minute until 11:18 A.M. What was the temperature at 11:18 A.M.?

78. ***Chemical Reaction.*** The temperature of a chemical compound was −5°C at 3:20 P.M. During a reaction, it increased 2°C per minute until 3:52 P.M. What was the temperature at 3:52 P.M.?

79. *Stock Price.* The price of a stock began the day at \$23.75 per share and dropped \$1.38 per hour for 8 hr. What was the price of the stock after 8 hr?

80. *Population Decrease.* The population of Bloomtown was 12,500. It decreased 380 each year for 4 yr. What was the population of the town after 4 yr?

81. *Diver's Position.* After diving 95 m below the sea level, a diver rises at a rate of 7 m/min for 9 min. Where is the diver in relation to the surface at the end of the 9-min period?

82. *Checking Account Balance.* Karen had \$68 in her checking account. After she had written checks to make seven purchases at \$13 each, what was the balance in her checking account?

83. *Drop in Temperature.* The temperature in Osgood was 62°F at 2:00 P.M. It dropped 6°F per hour for the next 4 hr. What was the temperature at the end of the 4-hr period?

84. *Juice Consumption.* Eliza bought a 64-oz container of cranberry juice and drank 8 oz per day for a week. How much juice was left in the container at the end of the week?

Skill Maintenance

85. Evaluate $\frac{x - 2y}{3}$ for $x = 20$ and $y = 7$. [1.1a]

86. Evaluate $\frac{d - e}{3d}$ for $d = 5$ and $e = 1$. [1.1a]

Subtract. [1.4a]

87. $-\frac{1}{2} - \left(-\frac{1}{6}\right)$

88. $8 - 12.3$

89. $31 - (-13)$

90. $-\frac{5}{12} - \left(-\frac{1}{3}\right)$

Write true or false. [1.2d]

91. $-10 > -12$

92. $0 \leq -1$

93. $4 < -8$

94. $-7 \geq -6$

Synthesis

95. If a is positive and b is negative, then $-ab$ is:

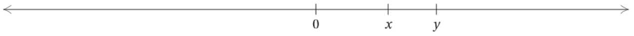
A. Positive.
B. Negative.
C. 0.
D. Cannot be determined without more information

96. If a is positive and b is negative, then $(-a)(-b)$ is:

A. Positive.
B. Negative.
C. 0.
D. Cannot be determined without more information

97. Below is a number line showing 0 and two positive numbers x and y. Use a compass or ruler to locate the following as best you can:

$2x, \quad 3x, \quad 2y, \quad -x, \quad -y, \quad x + y, \quad x - y, \quad x - 2y.$



98. Of all possible quotients of the numbers 10, $-\frac{1}{2}$, -5, and $\frac{1}{5}$, which two produce the largest quotient? Which two produce the smallest quotient?

1.6 Division of Real Numbers

We now consider division of real numbers. The definition of division results in rules for division that are the same as those for multiplication.

a Division of Integers

DIVISION

The quotient $a \div b$, or $\frac{a}{b}$, where $b \neq 0$, is that unique real number c for which $a = b \cdot c$.

Let's use the definition to divide integers.

EXAMPLES Divide, if possible. Check your answer.

1. $14 \div (-7) = -2$ *Think*: What number multiplied by -7 gives 14? That number is -2. *Check*: $(-2)(-7) = 14$.

2. $\frac{-32}{-4} = 8$ *Think*: What number multiplied by -4 gives -32? That number is 8. *Check*: $8(-4) = -32$.

3. $\frac{-10}{7} = -\frac{10}{7}$ *Think*: What number multiplied by 7 gives -10? That number is $-\frac{10}{7}$. *Check*: $-\frac{10}{7} \cdot 7 = -10$.

4. $\frac{-17}{0}$ is **not defined**. *Think*: What number multiplied by 0 gives -17? There is no such number because the product of 0 and *any* number is 0.

The rules for division are the same as those for multiplication.

To multiply or divide two real numbers (where the divisor is nonzero):

a) Multiply or divide the absolute values.
b) If the signs are the same, the answer is positive.
c) If the signs are different, the answer is negative.

Do Margin Exercises 1–6.

Excluding Division by 0

Example 4 shows why we cannot divide -17 by 0. We can use the same argument to show why we cannot divide any nonzero number b by 0. Consider $b \div 0$. We look for a number that when multiplied by 0 gives b. There is no such number because the product of 0 and any number is 0. Thus we cannot divide a nonzero number b by 0.

On the other hand, if we divide 0 by 0, we look for a number c such that $0 \cdot c = 0$. But $0 \cdot c = 0$ for any number c. Thus it appears that $0 \div 0$ could be any number we choose. Getting any answer we want when we divide 0 by 0 would be very confusing. Thus we agree that division by 0 is not defined.

OBJECTIVES

a Divide integers.

b Find the reciprocal of a real number.

c Divide real numbers.

d Solve applied problems involving division of real numbers.

SKILL TO REVIEW

Objective 1.5a: Multiply real numbers.

Multiply.

1. $-\frac{2}{9} \cdot \frac{4}{11}$ **2.** $\frac{13}{2} \cdot \left(-\frac{2}{25}\right)$

Divide.

1. $6 \div (-3)$

Think: What number multiplied by -3 gives 6?

2. $\frac{-15}{-3}$

Think: What number multiplied by -3 gives -15?

3. $-24 \div 8$

Think: What number multiplied by 8 gives -24?

4. $\frac{-48}{-6}$ **5.** $\frac{30}{-5}$

6. $\frac{30}{-7}$

Answers

Skill to Review:

1. $-\frac{8}{99}$ 2. $-\frac{13}{25}$

Margin Exercises:

1. -2 2. 5 3. -3 4. 8
5. -6 6. $-\frac{30}{7}$

EXCLUDING DIVISION BY 0

Division by 0 is not defined.

$$a \div 0, \text{ or } \frac{a}{0}, \text{ is not defined for all real numbers } a.$$

Dividing 0 by Other Numbers

Note that

$$0 \div 8 = 0 \text{ because } 0 = 0 \cdot 8; \qquad \frac{0}{-5} = 0 \text{ because } 0 = 0 \cdot (-5).$$

DIVIDENDS OF 0

Zero divided by any nonzero real number is 0:

$$\frac{0}{a} = 0; \qquad a \neq 0.$$

Divide, if possible.

7. $\frac{-5}{0}$ **8.** $\frac{0}{-3}$

EXAMPLES Divide.

5. $0 \div (-6) = 0$ **6.** $\frac{0}{12} = 0$ **7.** $\frac{-3}{0}$ is not defined.

Do Exercises 7 and 8.

b Reciprocals

When two numbers like $\frac{1}{2}$ and 2 are multiplied, the result is 1. Such numbers are called **reciprocals** of each other. Every nonzero real number has a reciprocal, also called a **multiplicative inverse**.

RECIPROCALS

Two numbers whose product is 1 are called **reciprocals**, or **multiplicative inverses**, of each other.

EXAMPLES Find the reciprocal.

8. $\frac{7}{8}$ The reciprocal of $\frac{7}{8}$ is $\frac{8}{7}$ because $\frac{7}{8} \cdot \frac{8}{7} = 1$.

9. -5 The reciprocal of -5 is $-\frac{1}{5}$ because $-5\left(-\frac{1}{5}\right) = 1$.

10. 3.9 The reciprocal of 3.9 is $\frac{1}{3.9}$ because $3.9\left(\frac{1}{3.9}\right) = 1$.

11. $-\frac{1}{2}$ The reciprocal of $-\frac{1}{2}$ is -2 because $\left(-\frac{1}{2}\right)(-2) = 1$.

12. $-\frac{2}{3}$ The reciprocal of $-\frac{2}{3}$ is $-\frac{3}{2}$ because $\left(-\frac{2}{3}\right)\left(-\frac{3}{2}\right) = 1$.

13. $\frac{3y}{8x}$ The reciprocal of $\frac{3y}{8x}$ is $\frac{8x}{3y}$ because $\left(\frac{3y}{8x}\right)\left(\frac{8x}{3y}\right) = 1$.

Answers

7. Not defined 8. 0

RECIPROCAL PROPERTIES

For $a \neq 0$, the reciprocal of a can be named $\frac{1}{a}$ and the reciprocal of $\frac{1}{a}$ is a.

The reciprocal of a nonzero number $\frac{a}{b}$ can be named $\frac{b}{a}$.

The number 0 has no reciprocal.

Do Exercises 9–14.

The reciprocal of a positive number is also a positive number, because the product of the two numbers must be the positive number 1. The reciprocal of a negative number is also a negative number, because the product of the two numbers must be the positive number 1.

THE SIGN OF A RECIPROCAL

The reciprocal of a number has the same sign as the number itself.

Caution!

It is important *not* to confuse *opposite* with *reciprocal*. Keep in mind that the opposite, or additive inverse, of a number is what we add to the number to get 0. The reciprocal, or multiplicative inverse, is what we multiply the number by to get 1.

Compare the following.

NUMBER	OPPOSITE (Change the sign.)	RECIPROCAL (Invert but do not change the sign.)
$-\frac{3}{8}$	$\frac{3}{8}$	$-\frac{8}{3}$
19	-19	$\frac{1}{19}$
$\frac{18}{7}$	$-\frac{18}{7}$	$\frac{7}{18}$
-7.9	7.9	$-\frac{1}{7.9}$, or $-\frac{10}{79}$
0	0	Not defined

$\left(-\frac{3}{8}\right)\left(-\frac{8}{3}\right) = 1$

$-\frac{3}{8} + \frac{3}{8} = 0$

Do Exercise 15.

Find the reciprocal.

9. $\frac{2}{3}$

10. $-\frac{5}{4}$

11. -3

12. $-\frac{1}{5}$

13. 1.3

14. $\frac{a}{6b}$

15. Complete the following table.

NUMBER	OPPOSITE	RECIPROCAL
$\frac{2}{3}$		
$-\frac{5}{4}$		
0		
1		
-8		
-4.7		

Answers

9. $\frac{3}{2}$ **10.** $-\frac{4}{5}$ **11.** $-\frac{1}{3}$ **12.** -5 **13.** $\frac{1}{1.3}$, or $\frac{10}{13}$ **14.** $\frac{6b}{a}$ **15.** $-\frac{2}{3}$ and $\frac{3}{2}$; $\frac{5}{4}$ and $-\frac{4}{5}$; 0 and not defined; -1 and 1; 8 and $-\frac{1}{8}$; 4.7 and $-\frac{1}{4.7}$, or $-\frac{10}{47}$

c Division of Real Numbers

We know that we can subtract by adding an opposite. Similarly, we can divide by multiplying by a reciprocal.

RECIPROCALS AND DIVISION

For any real numbers a and b, $b \neq 0$,

$$a \div b = \frac{a}{b} = a \cdot \frac{1}{b}.$$

(To divide, multiply by the reciprocal of the divisor.)

EXAMPLES Rewrite each division as a multiplication.

14. $-4 \div 3$ $\quad$ $-4 \div 3$ is the same as $-4 \cdot \frac{1}{3}$

15. $\frac{6}{-7}$ $\quad$ $\frac{6}{-7} = 6\left(-\frac{1}{7}\right)$

16. $\frac{3}{5} \div \left(-\frac{9}{7}\right)$ $\quad$ $\frac{3}{5} \div \left(-\frac{9}{7}\right) = \frac{3}{5}\left(-\frac{7}{9}\right)$

17. $\frac{x+2}{5}$ $\quad$ $\frac{x+2}{5} = (x+2)\frac{1}{5}$ $\quad$ Parentheses are necessary here.

18. $\frac{-17}{1/b}$ $\quad$ $\frac{-17}{1/b} = -17 \cdot b$

Do Exercises 16–20.

When actually doing division calculations, we sometimes multiply by a reciprocal and we sometimes divide directly. With fraction notation, it is usually better to multiply by a reciprocal. With decimal notation, it is usually better to divide directly.

EXAMPLES Divide by multiplying by the reciprocal of the divisor.

19. $\frac{2}{3} \div \left(-\frac{5}{4}\right) = \frac{2}{3} \cdot \left(-\frac{4}{5}\right) = -\frac{8}{15}$

20. $-\frac{5}{6} \div \left(-\frac{3}{4}\right) = -\frac{5}{6} \cdot \left(-\frac{4}{3}\right) = \frac{20}{18} = \frac{10 \cdot 2}{9 \cdot 2} = \frac{10}{9} \cdot \frac{2}{2} = \frac{10}{9}$

Caution!

Be careful *not* to change the sign when taking a reciprocal!

21. $-\frac{3}{4} \div \frac{3}{10} = -\frac{3}{4} \cdot \left(\frac{10}{3}\right) = -\frac{30}{12} = -\frac{5 \cdot 6}{2 \cdot 6} = -\frac{5}{2} \cdot \frac{6}{6} = -\frac{5}{2}$

Do Exercises 21 and 22.

Rewrite each division as a multiplication.

16. $\frac{4}{7} \div \left(-\frac{3}{5}\right)$

17. $\frac{5}{-8}$

18. $\frac{a-b}{7}$

19. $\frac{-23}{1/a}$

20. $-5 \div 7$

Divide by multiplying by the reciprocal of the divisor.

21. $\frac{4}{7} \div \left(-\frac{3}{5}\right)$

22. $-\frac{12}{7} \div \left(-\frac{3}{4}\right)$

Answers

16. $\frac{4}{7} \cdot \left(-\frac{5}{3}\right)$ **17.** $5 \cdot \left(-\frac{1}{8}\right)$
18. $(a-b) \cdot \frac{1}{7}$ **19.** $-23 \cdot a$
20. $-5 \cdot \left(\frac{1}{7}\right)$ **21.** $-\frac{20}{21}$ **22.** $\frac{16}{7}$

With decimal notation, it is easier to carry out long division than to multiply by the reciprocal.

EXAMPLES Divide.

22. $-27.9 \div (-3) = \dfrac{-27.9}{-3} = 9.3$ Do the long division $\overset{9.3}{3\overline{)27.9}}$.
The answer is positive.

23. $-6.3 \div 2.1 = -3$ Do the long division $\overset{3.}{2.1.\overline{)6.3_{\wedge}}}$
The answer is negative.

Do Exercises 23 and 24.

Divide.

23. $21.7 \div (-3.1)$

24. $-20.4 \div (-4)$

Consider the following:

1. $\dfrac{2}{3} = \dfrac{2}{3} \cdot 1 = \dfrac{2}{3} \cdot \dfrac{-1}{-1} = \dfrac{2(-1)}{3(-1)} = \dfrac{-2}{-3}$. Thus, $\dfrac{2}{3} = \dfrac{-2}{-3}$.

(A negative number divided by a negative number is positive.)

2. $-\dfrac{2}{3} = -1 \cdot \dfrac{2}{3} = \dfrac{-1}{1} \cdot \dfrac{2}{3} = \dfrac{-1 \cdot 2}{1 \cdot 3} = \dfrac{-2}{3}$. Thus, $-\dfrac{2}{3} = \dfrac{-2}{3}$.

(A negative number divided by a positive number is negative.)

3. $\dfrac{-2}{3} = \dfrac{-2}{3} \cdot 1 = \dfrac{-2}{3} \cdot \dfrac{-1}{-1} = \dfrac{-2(-1)}{3(-1)} = \dfrac{2}{-3}$. Thus, $-\dfrac{2}{3} = \dfrac{2}{-3}$.

(A positive number divided by a negative number is negative.)

We can use the following properties to make sign changes in fraction notation.

SIGN CHANGES IN FRACTION NOTATION

For any numbers a and b, $b \neq 0$:

1. $\dfrac{-a}{-b} = \dfrac{a}{b}$

(The opposite of a number a divided by the opposite of another number b is the same as the quotient of the two numbers a and b.)

2. $\dfrac{-a}{b} = \dfrac{a}{-b} = -\dfrac{a}{b}$

(The opposite of a number a divided by another number b is the same as the number a divided by the opposite of the number b, and both are the same as the opposite of *a divided by b*.)

Do Exercises 25–27.

Find two equal expressions for each number with negative signs in different places.

25. $\dfrac{-5}{6}$

26. $-\dfrac{8}{7}$

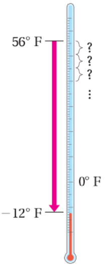
27. $\dfrac{10}{-3}$

Answers

23. -7 24. 5.1 25. $\dfrac{5}{-6}; -\dfrac{5}{6}$ 26. $\dfrac{8}{-7}; \dfrac{-8}{7}$
27. $\dfrac{-10}{3}; -\dfrac{10}{3}$

d Applications and Problem Solving

EXAMPLE 24 ***Chemical Reaction.*** During a chemical reaction, the temperature in a beaker decreased every minute by the same number of degrees. The temperature was 56°F at 10:10 A.M. By 10:42 A.M., the temperature had dropped to −12°F. By how many degrees did it change each minute?



We first determine by how many degrees d the temperature changed altogether. We subtract −12 from 56:

$$d = 56 - (-12) = 56 + 12 = 68.$$

The temperature changed a total of 68°. We can express this as −68° since the temperature dropped.

The amount of time t that passed was 42 − 10, or 32 min. Thus the number of degrees T that the temperature dropped each minute is given by

$$T = \frac{d}{t} = \frac{-68}{32} = -2.125.$$

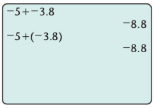
The change was −2.125°F per minute.

Do Exercise 28.

28. Chemical Reaction. During a chemical reaction, the temperature in a beaker decreased every minute by the same number of degrees. The temperature was 71°F at 2:12 P.M. By 2:37 P.M., the temperature had changed to −14°F. By how many degrees did it change each minute?

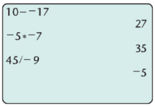
Calculator Corner

Operations on the Real Numbers We can perform operations on the real numbers on a graphing calculator. Recall that negative numbers are entered using the opposite key, (−), rather than the subtraction operation key, −. Consider the sum −5 + (−3.8). We use parentheses when we write this sum in order to separate the addition symbol and the "opposite of" symbol and thus make the expression more easily read. When we enter this calculation on a graphing calculator, however, the parentheses are not necessary. We can press (−) 5 + (−) 3 . 8 ENTER. The result is −8.8. Note that it is not incorrect to enter the parentheses. The result will be the same if this is done.



```
-5+-3.8
                 -8.8
-5+(-3.8)
                 -8.8
```

To find the difference 10 − (−17), we press 1 0 − (−) 1 7 ENTER. The result is 27. We can also multiply and divide real numbers. To find −5 · (−7), we press (−) 5 × (−) 7 ENTER, and to find 45 ÷ (−9), we press 4 5 ÷ (−) 9 ENTER. Note that it is not necessary to use parentheses in any of these calculations.



```
10--17
                   27
-5*-7
                   35
45/-9
                   -5
```

Exercises: Use a calculator to perform each operation.

1. $-8 + 4$
2. $1.2 + (-1.5)$
3. $-7 + (-5)$
4. $-7.6 + (-1.9)$
5. $-8 - 4$
6. $1.2 - (-1.5)$
7. $-7 - (-5)$
8. $-7.6 - (-1.9)$
9. $-8 \cdot 4$
10. $1.2 \cdot (-1.5)$
11. $-7 \cdot (-5)$
12. $-7.6 \cdot (-1.9)$
13. $-8 \div 4$
14. $1.2 \div (-1.5)$
15. $-7 \div (-5)$
16. $-7.6 \div (-1.9)$

Answer

28. −3.4°F per minute

1.6 Exercise Set

For Extra Help

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a Divide, if possible. Check each answer.

1. $48 \div (-6)$

2. $\frac{42}{-7}$

3. $\frac{28}{-2}$

4. $24 \div (-12)$

5. $\frac{-24}{8}$

6. $-18 \div (-2)$

7. $\frac{-36}{-12}$

8. $-72 \div (-9)$

9. $\frac{-72}{9}$

10. $\frac{-50}{25}$

11. $-100 \div (-50)$

12. $\frac{-200}{8}$

13. $-108 \div 9$

14. $\frac{-63}{-7}$

15. $\frac{200}{-25}$

16. $-300 \div (-16)$

17. $\frac{75}{0}$

18. $\frac{0}{-5}$

19. $\frac{0}{-2.6}$

20. $\frac{-23}{0}$

b Find the reciprocal.

21. $\frac{15}{7}$

22. $\frac{3}{8}$

23. $-\frac{47}{13}$

24. $-\frac{31}{12}$

25. 13

26. -10

27. -32

28. 15

29. $\frac{1}{-7.1}$

30. $\frac{1}{-4.9}$

31. $\frac{1}{9}$

32. $\frac{1}{16}$

33. $\frac{1}{4y}$

34. $\frac{-1}{8a}$

35. $\frac{2a}{3b}$

36. $\frac{-4y}{3x}$

C Rewrite each division as a multiplication.

37. $4 \div 17$

38. $5 \div (-8)$

39. $\frac{8}{-13}$

40. $-\frac{13}{47}$

41. $\frac{13.9}{-1.5}$

42. $-\frac{47.3}{21.4}$

43. $\frac{2}{3} \div \left(-\frac{4}{5}\right)$

44. $\frac{3}{4} \div \left(-\frac{7}{10}\right)$

45. $\frac{x}{\frac{1}{y}}$

46. $\frac{13}{\frac{1}{x}}$

47. $\frac{3x + 4}{5}$

48. $\frac{4y - 8}{-7}$

Divide.

49. $\frac{3}{4} \div \left(-\frac{2}{3}\right)$

50. $\frac{7}{8} \div \left(-\frac{1}{2}\right)$

51. $-\frac{5}{4} \div \left(-\frac{3}{4}\right)$

52. $-\frac{5}{9} \div \left(-\frac{5}{6}\right)$

53. $-\frac{2}{7} \div \left(-\frac{4}{9}\right)$

54. $-\frac{3}{5} \div \left(-\frac{5}{8}\right)$

55. $-\frac{3}{8} \div \left(-\frac{8}{3}\right)$

56. $-\frac{5}{8} \div \left(-\frac{6}{5}\right)$

57. $-\frac{5}{6} \div \frac{2}{3}$

58. $-\frac{7}{16} \div \frac{3}{8}$

59. $-\frac{9}{4} \div \frac{5}{12}$

60. $-\frac{3}{5} \div \frac{7}{10}$

61. $\frac{-11}{-13}$

62. $\frac{-21}{-25}$

63. $-6.6 \div 3.3$

64. $-44.1 \div (-6.3)$

65. $\frac{48.6}{-3}$

66. $\frac{-1.9}{20}$

67. $\frac{-12.5}{5}$

68. $\frac{-17.8}{3.2}$

69. $11.25 \div (-9)$

70. $-9.6 \div (-6.4)$

71. $\frac{-9}{17 - 17}$

72. $\frac{-8}{-5 + 5}$

d *Percent of Increase or Decrease in Employment.* A percent of increase is generally positive and a percent of decrease is generally negative. The table below lists estimates of the number of job opportunities for various occupations in 2006 and 2016. In Exercises 73–76, find the missing numbers.

	OCCUPATION	NUMBER OF JOBS IN 2006 (in thousands)	NUMBER OF JOBS IN 2016 (in thousands)	CHANGE	PERCENT OF INCREASE OR DECREASE
	Electrician	705	757	52	7.4%
	File clerk	234	137	−97	−41.5%
73.	Athletic trainer	17	21	4	
74.	Child-care worker	1388	1636	248	
75.	Cashier	3527	3411	−116	
76.	Fisherman	38	32	−6	

SOURCE: U.S. Bureau of Labor Statistics *Occupational Outlook Handbook*

Skill Maintenance

Simplify.

77. $\frac{1}{4} - \frac{1}{2}$ [1.4a]

78. $-9 - 3 + 17$ [1.4a]

79. $35 \cdot (-1.2)$ [1.5a]

80. $4 \cdot (-6) \cdot (-2) \cdot (-1)$ [1.5a]

81. $13.4 + (-4.9)$ [1.3a]

82. $-\frac{3}{8} - \left(-\frac{1}{4}\right)$ [1.4a]

Convert to decimal notation. [1.2c]

83. $-\frac{1}{11}$

84. $\frac{11}{12}$

85. $\frac{15}{4}$

86. $-\frac{10}{3}$

Synthesis

87. Find the reciprocal of -10.5. What happens if you take the reciprocal of the result?

88. Determine those real numbers a for which the opposite of a is the same as the reciprocal of a.

Determine whether each expression represents a positive number or a negative number when a and b are negative.

89. $\frac{-a}{b}$

90. $\frac{-a}{-b}$

91. $-\left(\frac{a}{-b}\right)$

92. $-\left(\frac{-a}{b}\right)$

93. $-\left(\frac{-a}{-b}\right)$

1.7

Properties of Real Numbers

OBJECTIVES

- **a** Find equivalent fraction expressions and simplify fraction expressions.
- **b** Use the commutative and associative laws to find equivalent expressions.
- **c** Use the distributive laws to multiply expressions like 8 and $x - y$.
- **d** Use the distributive laws to factor expressions like $4x - 12 + 24y$.
- **e** Collect like terms.

SKILL TO REVIEW

Objective 1.3a: Add real numbers.

Add.

1. $-16 + 5$ **2.** $29 + (-23)$

a Equivalent Expressions

In solving equations and doing other kinds of work in algebra, we manipulate expressions in various ways. For example, instead of $x + x$, we might write $2x$, knowing that the two expressions represent the same number for any allowable replacement of x. In that sense, the expressions $x + x$ and $2x$ are **equivalent**, as are $\frac{3}{x}$ and $\frac{3x}{x^2}$, even though 0 is not an allowable replacement because division by 0 is not defined.

EQUIVALENT EXPRESSIONS

Two expressions that have the same value for all allowable replacements are called **equivalent**.

The expressions $x + 3x$ and $5x$ are *not* equivalent, as we see in Margin Exercise 2.

Do Exercises 1 and 2.

In this section, we will consider several laws of real numbers that will allow us to find equivalent expressions. The first two laws are the *identity properties of 0 and 1*.

THE IDENTITY PROPERTY OF 0

For any real number a,

$$a + 0 = 0 + a = a.$$

(The number 0 is the *additive identity*.)

THE IDENTITY PROPERTY OF 1

For any real number a,

$$a \cdot 1 = 1 \cdot a = a.$$

(The number 1 is the *multiplicative identity*.)

We often refer to the use of the identity property of 1 as "multiplying by 1." We can use this method to find equivalent fraction expressions. Recall from arithmetic that to multiply with fraction notation, we multiply the numerators and multiply the denominators.

EXAMPLE 1 Write a fraction expression equivalent to $\frac{2}{3}$ with a denominator of $3x$:

$$\frac{2}{3} = \frac{\square}{3x}.$$

Complete the table by evaluating each expression for the given values.

1.

Value	$x + x$	$2x$
$x = 3$		
$x = -6$		
$x = 4.8$		

2.

Value	$x + 3x$	$5x$
$x = 2$		
$x = -6$		
$x = 4.8$		

Answers

Skill to Review:

1. -11 **2.** 6

Margin Exercises:

1. 6, 6; -12, -12; 9.6, 9.6 **2.** 8, 10; -24, -30; 19.2, 24

Note that $3x = 3 \cdot x$. We want fraction notation for $\frac{2}{3}$ that has a denominator of $3x$, but the denominator 3 is missing a factor of x. Thus we multiply by 1, using x/x as an equivalent expression for 1:

$$\frac{2}{3} = \frac{2}{3} \cdot 1 = \frac{2}{3} \cdot \frac{x}{x} = \frac{2x}{3x}.$$

The expressions $2/3$ and $2x/(3x)$ are equivalent. They have the same value for any allowable replacement. Note that $2x/(3x)$ is not defined for a replacement of 0, but for all nonzero real numbers, the expressions $2/3$ and $2x/(3x)$ have the same value.

Do Exercises 3 and 4.

In algebra, we consider an expression like $2/3$ to be "simplified" from $2x/(3x)$. To find such simplified expressions, we use the identity property of 1 to remove a factor of 1.

EXAMPLE 2 Simplify: $-\frac{20x}{12x}$.

$$-\frac{20x}{12x} = -\frac{5 \cdot 4x}{3 \cdot 4x}$$

We look for the largest factor common to both the numerator and the denominator and factor each.

$$= -\frac{5}{3} \cdot \frac{4x}{4x}$$

Factoring the fraction expression

$$= -\frac{5}{3} \cdot 1$$

$\frac{4x}{4x} = 1$

$$= -\frac{5}{3}$$

Removing a factor of 1 using the identity property of 1

EXAMPLE 3 Simplify: $\frac{14ab}{56a}$.

$$\frac{14ab}{56a} = \frac{14a \cdot b}{14a \cdot 4} = \frac{14a}{14a} \cdot \frac{b}{4} = 1 \cdot \frac{b}{4} = \frac{b}{4}$$

Do Exercises 5–8.

b The Commutative and Associative Laws

The Commutative Laws

Let's examine the expressions $x + y$ and $y + x$, as well as xy and yx.

EXAMPLE 4 Evaluate $x + y$ and $y + x$ when $x = 4$ and $y = 3$.

We substitute 4 for x and 3 for y in both expressions:

$$x + y = 4 + 3 = 7; \qquad y + x = 3 + 4 = 7.$$

EXAMPLE 5 Evaluate xy and yx when $x = 3$ and $y = -12$.

We substitute 3 for x and -12 for y in both expressions:

$$xy = 3 \cdot (-12) = -36; \qquad yx = (-12) \cdot 3 = -36.$$

Do Exercises 9 and 10.

3. Write a fraction expression equivalent to $\frac{3}{4}$ with a denominator of 8:

$$\frac{3}{4} = \frac{\square}{8}.$$

4. Write a fraction expression equivalent to $\frac{3}{4}$ with a denominator of $4t$:

$$\frac{3}{4} = \frac{\square}{4t}.$$

Simplify.

5. $\frac{3y}{4y}$

6. $-\frac{16m}{12m}$

7. $\frac{5xy}{40y}$

8. $\frac{18p}{24pq}$

9. Evaluate $x + y$ and $y + x$ when $x = -2$ and $y = 3$.

10. Evaluate xy and yx when $x = -2$ and $y = 5$.

Answers

3. $\frac{6}{8}$ **4.** $\frac{3t}{4t}$ **5.** $\frac{3}{4}$ **6.** $-\frac{4}{3}$

7. $\frac{x}{8}$ **8.** $\frac{3}{4q}$ **9.** 1; 1 **10.** -10; -10

The expressions $x + y$ and $y + x$ have the same values no matter what the variables stand for. Thus they are equivalent. Therefore, when we add two numbers, the order in which we add does not matter. Similarly, the expressions xy and yx are equivalent. They also have the same values, no matter what the variables stand for. Therefore, when we multiply two numbers, the order in which we multiply does not matter.

The following are examples of general patterns or laws.

THE COMMUTATIVE LAWS

Addition. For any numbers a and b,

$$a + b = b + a.$$

(We can change the order when adding without affecting the answer.)

Multiplication. For any numbers a and b,

$$ab = ba.$$

(We can change the order when multiplying without affecting the answer.)

Using a commutative law, we know that $x + 2$ and $2 + x$ are equivalent. Similarly, $3x$ and $x(3)$ are equivalent. Thus, in an algebraic expression, we can replace one with the other and the result will be equivalent to the original expression.

EXAMPLE 6 Use the commutative laws to write an equivalent expression: **(a)** $y + 5$; **(b)** mn; **(c)** $7 + xy$.

a) An expression equivalent to $y + 5$ is $5 + y$ by the commutative law of addition.

b) An expression equivalent to mn is nm by the commutative law of multiplication.

c) An expression equivalent to $7 + xy$ is $xy + 7$ by the commutative law of addition. Another expression equivalent to $7 + xy$ is $7 + yx$ by the commutative law of multiplication. Another equivalent expression is $yx + 7$.

Do Exercises 11–13.

The Associative Laws

Now let's examine the expressions $a + (b + c)$ and $(a + b) + c$. Note that these expressions involve the use of parentheses as *grouping* symbols, and they also involve three numbers. Calculations within parentheses are to be done first.

EXAMPLE 7 Calculate and compare: $3 + (8 + 5)$ and $(3 + 8) + 5$.

$3 + (8 + 5) = 3 + 13$ Calculating within parentheses first; adding the 8 and the 5

$= 16;$

$(3 + 8) + 5 = 11 + 5$ Calculating within parentheses first; adding the 3 and the 8

$= 16$

STUDY TIPS

VIDEO RESOURCES

Developed and produced especially for this text, and now available on DVD-ROM with Chapter Test Prep Videos that walk you through step-by-step solutions to all the Chapter Test exercises, these videos feature an engaging team of instructors who present material and concepts using examples and exercises from every section of the text. The complete digitized video set, both affordable and portable, makes it easy and convenient for you to watch video segments at home or on campus. (ISBN: 978-0-321-61365-5, available for purchase at www.MyPearsonStore.com)

Use a commutative law to write an equivalent expression.

11. $x + 9$

12. pq

13. $xy + t$

Answers

11. $9 + x$ 12. qp
13. $t + xy$, or $yx + t$, or $t + yx$

The two expressions in Example 7 name the same number. Moving the parentheses to group the additions differently does not affect the value of the expression.

EXAMPLE 8 Calculate and compare: $3 \cdot (4 \cdot 2)$ and $(3 \cdot 4) \cdot 2$.

$$3 \cdot (4 \cdot 2) = 3 \cdot 8 = 24; \qquad (3 \cdot 4) \cdot 2 = 12 \cdot 2 = 24$$

Do Exercises 14 and 15.

14. Calculate and compare:
$8 + (9 + 2)$ and $(8 + 9) + 2$.

15. Calculate and compare:
$10 \cdot (5 \cdot 3)$ and $(10 \cdot 5) \cdot 3$.

You may have noted that when only addition is involved, numbers can be grouped any way we please without affecting the answer. When only multiplication is involved, numbers can also be grouped any way we please without affecting the answer.

THE ASSOCIATIVE LAWS

Addition. For any numbers a, b, and c,

$$a + (b + c) = (a + b) + c.$$

(Numbers can be grouped in any manner for addition.)

Multiplication. For any numbers a, b, and c,

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c.$$

(Numbers can be grouped in any manner for multiplication.)

EXAMPLE 9 Use an associative law to write an equivalent expression: **(a)** $(y + z) + 3$; **(b)** $8(xy)$.

a) An expression equivalent to $(y + z) + 3$ is $y + (z + 3)$ by the associative law of addition.

b) An expression equivalent to $8(xy)$ is $(8x)y$ by the associative law of multiplication.

Do Exercises 16 and 17.

Use an associative law to write an equivalent expression.

16. $r + (s + 7)$

17. $9(ab)$

The associative laws say that numbers can be grouped any way we please when only additions or only multiplications are involved. Thus we often omit the parentheses. For example,

$$x + (y + 2) \quad \text{means} \quad x + y + 2, \qquad \text{and} \qquad (lw)h \quad \text{means} \quad lwh.$$

Using the Commutative and Associative Laws Together

EXAMPLE 10 Use the commutative and associative laws to write at least three expressions equivalent to $(x + 5) + y$.

a) $(x + 5) + y = x + (5 + y)$ Using the associative law first and then using the commutative law
$= x + (y + 5)$

b) $(x + 5) + y = y + (x + 5)$ Using the commutative law twice
$= y + (5 + x)$

c) $(x + 5) + y = (5 + x) + y$ Using the commutative law first and then the associative law
$= 5 + (x + y)$

Answers

14. 19; 19 **15.** 150; 150 **16.** $(r + s) + 7$
17. $(9a)b$

EXAMPLE 11 Use the commutative and associative laws to write at least three expressions equivalent to $(3x)y$.

a) $(3x)y = 3(xy)$ Using the associative law first and then using the commutative law
$= 3(yx)$

b) $(3x)y = y(3x)$ Using the commutative law twice
$= y(x \cdot 3)$

c) $(3x)y = (x \cdot 3)y$ Using the commutative law, and then the associative law, and then the commutative law again
$= x(3y)$
$= x(y \cdot 3)$

Do Exercises 18 and 19.

Use the commutative and associative laws to write at least three equivalent expressions.

18. $4(tu)$

19. $r + (2 + s)$

C The Distributive Laws

The *distributive laws* are the basis of many procedures in both arithmetic and algebra. They are probably the most important laws that we use to manipulate algebraic expressions. The distributive law of multiplication over addition involves two operations: addition and multiplication.

Let's begin by considering a multiplication problem from arithmetic:

$$\begin{array}{r l} 4\;5 & \\ 7 & \\ \hline 3\;5 & \leftarrow \text{This is } 7 \cdot 5. \\ \underline{2\;8\;0} & \leftarrow \text{This is } 7 \cdot 40. \\ 3\;1\;5 & \leftarrow \text{This is the sum } 7 \cdot 5 + 7 \cdot 40. \end{array}$$

To carry out the multiplication, we actually added two products. That is,

$$7 \cdot 45 = 7(5 + 40) = 7 \cdot 5 + 7 \cdot 40.$$

Let's examine this further. If we wish to multiply a sum of several numbers by a factor, we can either add and then multiply, or multiply and then add.

EXAMPLE 12 Compute in two ways: $5 \cdot (4 + 8)$.

a) $5 \cdot \underbrace{(4 + 8)}_{\downarrow}$ Adding within parentheses first, and then multiplying

$= 5 \cdot 12$

$= 60$

b) $5 \cdot (4 + 8) = \underbrace{(5 \cdot 4)}_{\downarrow} + \underbrace{(5 \cdot 8)}_{\downarrow}$ Distributing the multiplication to terms within parentheses first and then adding

$= 20 + 40$

$= 60$

Do Exercises 20–22.

Compute.

20. a) $7 \cdot (3 + 6)$
b) $(7 \cdot 3) + (7 \cdot 6)$

21. a) $2 \cdot (10 + 30)$
b) $(2 \cdot 10) + (2 \cdot 30)$

22. a) $(2 + 5) \cdot 4$
b) $(2 \cdot 4) + (5 \cdot 4)$

THE DISTRIBUTIVE LAW OF MULTIPLICATION OVER ADDITION

For any numbers a, b, and c,

$$a(b + c) = ab + ac.$$

Answers

18. $(4t)u, (tu)4, t(4u)$; answers may vary
19. $(2 + r) + s, (r + s) + 2, s + (r + 2)$; answers may vary
20. (a) $7 \cdot 9 = 63$; (b) $21 + 42 = 63$
21. (a) $2 \cdot 40 = 80$; (b) $20 + 60 = 80$
22. (a) $7 \cdot 4 = 28$; (b) $8 + 20 = 28$

In the statement of the distributive law, we know that in an expression such as $ab + ac$, the multiplications are to be done first according to the rules for order of operations. So, instead of writing $(4 \cdot 5) + (4 \cdot 7)$, we can write $4 \cdot 5 + 4 \cdot 7$. However, in $a(b + c)$, we cannot omit the parentheses. If we did, we would have $ab + c$, which means $(ab) + c$. For example, $3(4 + 2) = 3(6) = 18$, but $3 \cdot 4 + 2 = 12 + 2 = 14$.

There is another distributive law that relates multiplication and subtraction. This law says that to multiply by a difference, we can either subtract and then multiply, or multiply and then subtract.

THE DISTRIBUTIVE LAW OF MULTIPLICATION OVER SUBTRACTION

For any numbers a, b, and c,

$$a(b - c) = ab - ac.$$

We often refer to "*the* distributive law" when we mean *either* or *both* of these laws.

Do Exercises 23–25.

Calculate.

23. a) $4(5 - 3)$

b) $4 \cdot 5 - 4 \cdot 3$

24. a) $-2 \cdot (5 - 3)$

b) $-2 \cdot 5 - (-2) \cdot 3$

25. a) $5 \cdot (2 - 7)$

b) $5 \cdot 2 - 5 \cdot 7$

What do we mean by the *terms* of an expression? **Terms** are separated by addition signs. If there are subtraction signs, we can find an equivalent expression that uses addition signs.

EXAMPLE 13 What are the terms of $3x - 4y + 2z$?

We have

$3x - 4y + 2z = 3x + (-4y) + 2z.$ Separating parts with + signs

The terms are $3x$, $-4y$, and $2z$.

Do Exercises 26 and 27.

What are the terms of each expression?

26. $5x - 8y + 3$

27. $-4y - 2x + 3z$

The distributive laws are a basis for a procedure in algebra called **multiplying**. In an expression like $8(a + 2b - 7)$, we multiply each term inside the parentheses by 8:

$$8(a + 2b - 7) = 8 \cdot a + 8 \cdot 2b - 8 \cdot 7 = 8a + 16b - 56.$$

EXAMPLES Multiply.

14. $9(x - 5) = 9 \cdot x - 9 \cdot 5$ Using the distributive law of multiplication over subtraction

$= 9x - 45$

15. $\frac{2}{3}(w + 1) = \frac{2}{3} \cdot w + \frac{2}{3} \cdot 1$ Using the distributive law of multiplication over addition

$= \frac{2}{3}w + \frac{2}{3}$

16. $\frac{4}{3}(s - t + w) = \frac{4}{3}s - \frac{4}{3}t + \frac{4}{3}w$ Using both distributive laws

Do Exercises 28–30.

Multiply.

28. $3(x - 5)$

29. $5(x + 1)$

30. $\frac{3}{5}(p + q - t)$

Answers

23. (a) $4 \cdot 2 = 8$; (b) $20 - 12 = 8$
24. (a) $-2 \cdot 2 = -4$; (b) $-10 + 6 = -4$
25. (a) $5(-5) = -25$; (b) $10 - 35 = -25$
26. $5x, -8y, 3$ **27.** $-4y, -2x, 3z$
28. $3x - 15$ **29.** $5x + 5$
30. $\frac{3}{5}p + \frac{3}{5}q - \frac{3}{5}t$

EXAMPLE 17 Multiply: $-4(x - 2y + 3z)$.

$$\begin{aligned} -4(x - 2y + 3z) &= -4 \cdot x - (-4)(2y) + (-4)(3z) && \text{Using both distributive laws} \\ &= -4x - (-8y) + (-12z) && \text{Multiplying} \\ &= -4x + 8y - 12z \end{aligned}$$

We can also do this problem by first finding an equivalent expression with all plus signs and then multiplying:

$$\begin{aligned} -4(x - 2y + 3z) &= -4[x + (-2y) + 3z] \\ &= -4 \cdot x + (-4)(-2y) + (-4)(3z) \\ &= -4x + 8y - 12z. \end{aligned}$$

Do Exercises 31–33.

Multiply.

31. $-2(x - 3)$

32. $5(x - 2y + 4z)$

33. $-5(x - 2y + 4z)$

EXAMPLES Name the property or law illustrated by each equation.

Equation	*Property*
18. $5x = x(5)$	Commutative law of multiplication
19. $a + (8.5 + b) = (a + 8.5) + b$	Associative law of addition
20. $0 + 11 = 11$	Identity property of 0
21. $(-5s)t = -5(st)$	Associative law of multiplication
22. $\frac{3}{4} \cdot 1 = \frac{3}{4}$	Identity property of 1
23. $12.5(w - 3) = 12.5w - 12.5(3)$	Distributive law of multiplication over subtraction
24. $y + \frac{1}{2} = \frac{1}{2} + y$	Commutative law of addition

Do Exercises 34–40.

Name the property or law illustrated by each equation.

34. $(-8a)b = -8(ab)$

35. $p \cdot 1 = p$

36. $m + 34 = 34 + m$

37. $2(t + 5) = 2t + 2(5)$

38. $0 + k = k$

39. $-8x = x(-8)$

40. $x + (4.3 + b) = (x + 4.3) + b$

d Factoring

Factoring is the reverse of multiplying. To factor, we can use the distributive laws in reverse:

$$ab + ac = a(b + c) \quad \text{and} \quad ab - ac = a(b - c).$$

FACTORING

To **factor** an expression is to find an equivalent expression that is a product.

To factor $9x - 45$, for example, we find an equivalent expression that is a product: $9(x - 5)$. This reverses the multiplication that we did in Example 14. When all the terms of an expression have a factor in common, we can "factor it out" using the distributive laws. Note the following.

$9x$ has the factors $9, -9, 3, -3, 1, -1, x, -x, 3x, -3x, 9x, -9x$;

-45 has the factors $1, -1, 3, -3, 5, -5, 9, -9, 15, -15, 45, -45$

Answers

31. $-2x + 6$ **32.** $5x - 10y + 20z$
33. $-5x + 10y - 20z$ **34.** Associative law of multiplication **35.** Identity property of 1
36. Commutative law of addition
37. Distributive law of multiplication over addition **38.** Identity property of 0
39. Commutative law of multiplication
40. Associative law of addition

We generally remove the largest common factor. In this case, that factor is 9. Thus,

$$\begin{aligned} 9x - 45 &= 9 \cdot x - 9 \cdot 5 \\ &= 9(x - 5). \end{aligned}$$

Remember that an expression has been factored when we have found an equivalent expression that is a product. Above, we note that $9x - 45$ and $9(x - 5)$ are equivalent expressions. The expression $9x - 45$ is the difference of $9x$ and 45; the expression $9(x - 5)$ is the product of 9 and $(x - 5)$.

EXAMPLES Factor.

25. $5x - 10 = 5 \cdot x - 5 \cdot 2$ Try to do this step mentally.

$= 5(x - 2)$ You can check by multiplying.

26. $ax - ay + az = a(x - y + z)$

27. $9x + 27y - 9 = 9 \cdot x + 9 \cdot 3y - 9 \cdot 1 = 9(x + 3y - 1)$

Note in Example 27 that you might, at first, just factor out a 3, as follows:

$$\begin{aligned} 9x + 27y - 9 &= 3 \cdot 3x + 3 \cdot 9y - 3 \cdot 3 \\ &= 3(3x + 9y - 3). \end{aligned}$$

At this point, the mathematics is correct, but the answer is not because there is another factor of 3 that can be factored out, as follows:

$$\begin{aligned} 3 \cdot 3x + 3 \cdot 9y - 3 \cdot 3 &= 3(3x + 9y - 3) \\ &= 3(3 \cdot x + 3 \cdot 3y - 3 \cdot 1) \\ &= 3 \cdot 3(x + 3y - 1) \\ &= 9(x + 3y - 1). \end{aligned}$$

We now have a correct answer, but it took more work than we did in Example 27. Thus it is better to look for the *greatest common factor* at the outset.

EXAMPLES Factor. Try to write just the answer, if you can.

28. $5x - 5y = 5(x - y)$

29. $-3x + 6y - 9z = -3(x - 2y + 3z)$

We usually factor out a negative factor when the first term is negative. The way we factor can depend on the situation in which we are working. We might also factor the expression in Example 29 as follows:

$$-3x + 6y - 9z = 3(-x + 2y - 3z).$$

30. $18z - 12x - 24 = 6(3z - 2x - 4)$

31. $\frac{1}{2}x + \frac{3}{2}y - \frac{1}{2} = \frac{1}{2}(x + 3y - 1)$

Remember that you can always check factoring by multiplying. Keep in mind that an expression is factored when it is written as a product.

Do Exercises 41–46.

Factor.

41. $6x - 12$

42. $3x - 6y + 9$

43. $bx + by - bz$

44. $16a - 36b + 42$

45. $\frac{3}{8}x - \frac{5}{8}y + \frac{7}{8}$

46. $-12x + 32y - 16z$

Answers

41. $6(x - 2)$ 42. $3(x - 2y + 3)$
43. $b(x + y - z)$ 44. $2(8a - 18b + 21)$
45. $\frac{1}{8}(3x - 5y + 7)$ 46. $-4(3x - 8y + 4z)$, or $4(-3x + 8y - 4z)$

e Collecting Like Terms

Terms such as $5x$ and $-4x$, whose variable factors are exactly the same, are called **like terms.** Similarly, numbers, such as -7 and 13, are like terms. Also, $3y^2$ and $9y^2$ are like terms because the variables are raised to the same power. Terms such as $4y$ and $5y^2$ are not like terms, and $7x$ and $2y$ are not like terms.

The process of **collecting like terms** is also based on the distributive laws. We can apply a distributive law when a factor is on the right because of the commutative law of multiplication.

Later in this text, terminology like "collecting like terms" and "combining like terms" will also be referred to as "simplifying."

EXAMPLES Collect like terms. Try to write just the answer, if you can.

32. $4x + 2x = (4 + 2)x = 6x$ Factoring out the x using a distributive law

33. $2x + 3y - 5x - 2y = 2x - 5x + 3y - 2y$
$= (2 - 5)x + (3 - 2)y = -3x + 1y = -3x + y$

34. $3x - x = 3x - 1x = (3 - 1)x = 2x$

35. $x - 0.24x = 1 \cdot x - 0.24x = (1 - 0.24)x = 0.76x$

36. $x - 6x = 1 \cdot x - 6 \cdot x = (1 - 6)x = -5x$

37. $4x - 7y + 9x - 5 + 3y - 8 = 13x - 4y - 13$

38.
$$\begin{aligned}\tfrac{2}{3}a - b + \tfrac{4}{5}a + \tfrac{1}{4}b - 10 &= \tfrac{2}{3}a - 1 \cdot b + \tfrac{4}{5}a + \tfrac{1}{4}b - 10\\ &= \left(\tfrac{2}{3} + \tfrac{4}{5}\right)a + \left(-1 + \tfrac{1}{4}\right)b - 10\\ &= \left(\tfrac{10}{15} + \tfrac{12}{15}\right)a + \left(-\tfrac{4}{4} + \tfrac{1}{4}\right)b - 10\\ &= \tfrac{22}{15}a - \tfrac{3}{4}b - 10\end{aligned}$$

Do Exercises 47–53.

Collect like terms.

47. $6x - 3x$

48. $7x - x$

49. $x - 9x$

50. $x - 0.41x$

51. $5x + 4y - 2x - y$

52. $3x - 7x - 11 + 8y + 4 - 13y$

53. $-\dfrac{2}{3} - \dfrac{3}{5}x + y + \dfrac{7}{10}x - \dfrac{2}{9}y$

STUDY TIPS

LEARNING RESOURCES

Please see the preface for more information on these resources and others. To order any of our products, call (800) 824-7799 in the United States or (201) 767-5021 outside the United States, or visit your campus bookstore.

- The *Student's Solutions Manual* contains fully worked-out solutions to the odd-numbered exercises in the exercise sets, as well as solutions to all exercises in the Mid-Chapter Reviews, end-of-chapter Review Exercises, Chapter Tests, and Cumulative Reviews. (ISBN: 978-0-321-61362-2)
- *Worksheets for Classroom or Lab Practice* provide a list of learning objectives, vocabulary and practice problems, and extra practice problems with ample work space. (ISBN: 978-0-321-61368-4)
- As described on p. 56 and in the Preface, Video Resources on DVD Featuring Chapter Test Prep Videos provide section-level lectures for every objective and step-by-step solutions to all the Chapter Test exercises in this textbook. The Chapter Test videos are also available on YouTube (search using BittingerIntroInter) and in MyMathLab.
- InterAct Math Tutorial Website (www.interactmath.com) provides algorithmically generated practice exercises that correlate directly to the exercises in the textbook.
- MathXL® Tutorials on CD provide practice exercises correlated at the objective level to the exercises in the textbook. Every practice exercise is accompanied by an example and a guided solution, and selected exercises may also include a video clip to help illustrate a concept.

Answers

47. $3x$ **48.** $6x$ **49.** $-8x$ **50.** $0.59x$
51. $3x + 3y$ **52.** $-4x - 5y - 7$
53. $\dfrac{1}{10}x + \dfrac{7}{9}y - \dfrac{2}{3}$

1.7 Exercise Set

For Extra Help

MyMathLab

MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

a Find an equivalent expression with the given denominator.

1. $\frac{3}{5} = \frac{\square}{5y}$

2. $\frac{5}{8} = \frac{\square}{8t}$

3. $\frac{2}{3} = \frac{\square}{15x}$

4. $\frac{6}{7} = \frac{\square}{14y}$

5. $\frac{2}{x} = \frac{\square}{x^2}$

6. $\frac{4}{9x} = \frac{\square}{9xy}$

Simplify.

7. $-\frac{24a}{16a}$

8. $-\frac{42t}{18t}$

9. $-\frac{42ab}{36ab}$

10. $-\frac{64pq}{48pq}$

11. $\frac{20st}{15t}$

12. $\frac{21w}{7wz}$

b Write an equivalent expression. Use a commutative law.

13. $y + 8$

14. $x + 3$

15. mn

16. yz

17. $9 + xy$

18. $11 + ab$

19. $ab + c$

20. $rs + t$

Write an equivalent expression. Use an associative law.

21. $a + (b + 2)$

22. $3(vw)$

23. $(8x)y$

24. $(y + z) + 7$

25. $(a + b) + 3$

26. $(5 + x) + y$

27. $3(ab)$

28. $(6x)y$

Use the commutative and associative laws to write three equivalent expressions.

29. $(a + b) + 2$

30. $(3 + x) + y$

31. $5 + (v + w)$

32. $6 + (x + y)$

33. $(xy)3$

34. $(ab)5$

35. $7(ab)$

36. $5(xy)$

c Multiply.

37. $2(b + 5)$

38. $4(x + 3)$

39. $7(1 + t)$

40. $4(1 + y)$

41. $6(5x + 2)$

42. $9(6m + 7)$

43. $7(x + 4 + 6y)$

44. $4(5x + 8 + 3p)$

45. $7(x - 3)$

46. $15(y - 6)$

47. $-3(x - 7)$

48. $1.2(x - 2.1)$

49. $\frac{2}{3}(b - 6)$

50. $\frac{5}{8}(y + 16)$

51. $7.3(x - 2)$

52. $5.6(x - 8)$

53. $-\frac{3}{5}(x - y + 10)$

54. $-\frac{2}{3}(a + b - 12)$

55. $-9(-5x - 6y + 8)$

56. $-7(-2x - 5y + 9)$

57. $-4(x - 3y - 2z)$

58. $8(2x - 5y - 8z)$

59. $3.1(-1.2x + 3.2y - 1.1)$

60. $-2.1(-4.2x - 4.3y - 2.2)$

List the terms of each expression.

61. $4x + 3z$

62. $8x - 1.4y$

63. $7x + 8y - 9z$

64. $8a + 10b - 18c$

d Factor. Check by multiplying.

65. $2x + 4$

66. $5y + 20$

67. $30 + 5y$

68. $7x + 28$

69. $14x + 21y$

70. $18a + 24b$

71. $14t - 7$

72. $25m - 5$

73. $8x - 24$

74. $10x - 50$

75. $18a - 24b$

76. $32x - 20y$

77. $-4y + 32$

78. $-6m + 24$

79. $5x + 10 + 15y$

80. $9a + 27b + 81$

81. $16m - 32n + 8$

82. $6x + 10y - 2$

83. $12a + 4b - 24$

84. $8m - 4n + 12$

85. $8x + 10y - 22$

86. $9a + 6b - 15$

87. $ax - a$

88. $by - 9b$

89. $ax - ay - az$

90. $cx + cy - cz$

91. $-18x + 12y + 6$

92. $-14x + 21y + 7$

93. $\frac{2}{3}x - \frac{5}{3}y + \frac{1}{3}$

94. $\frac{3}{5}a + \frac{4}{5}b - \frac{1}{5}$

95. $36x - 6y + 18z$

96. $8a - 4b + 20c$

e Collect like terms.

97. $9a + 10a$

98. $12x + 2x$

99. $10a - a$

100. $-16x + x$

101. $2x + 9z + 6x$

102. $3a - 5b + 7a$

103. $7x + 6y^2 + 9y^2$

104. $12m^2 + 6q + 9m^2$

105. $41a + 90 - 60a - 2$

106. $42x - 6 - 4x + 2$

107. $23 + 5t + 7y - t - y - 27$

108. $45 - 90d - 87 - 9d + 3 + 7d$

109. $\frac{1}{2}b + \frac{1}{2}b$

110. $\frac{2}{3}x + \frac{1}{3}x$

111. $2y + \frac{1}{4}y + y$

112. $\frac{1}{2}a + a + 5a$

113. $11x - 3x$

114. $9t - 17t$

115. $6n - n$

116. $100t - t$

117. $y - 17y$

118. $3m - 9m + 4$

119. $-8 + 11a - 5b + 6a - 7b + 7$

120. $8x - 5x + 6 + 3y - 2y - 4$

121. $9x + 2y - 5x$

122. $8y - 3z + 4y$

123. $11x + 2y - 4x - y$

124. $13a + 9b - 2a - 4b$

125. $2.7x + 2.3y - 1.9x - 1.8y$

126. $6.7a + 4.3b - 4.1a - 2.9b$

127. $\frac{13}{2}a + \frac{9}{5}b - \frac{2}{3}a - \frac{3}{10}b - 42$

128. $\frac{11}{4}x + \frac{2}{3}y - \frac{4}{5}x - \frac{1}{6}y + 12$

Skill Maintenance

Simplify.

129. $-28 - (-2)$ [1.4a]

130. $-200 + 85$ [1.3a]

131. $-16(-10)$ [1.5a]

132. $-88 \div (-11)$ [1.6a]

133. $\frac{400}{-80}$ [1.6a]

134. $38 - (-12)$ [1.4a]

135. Evaluate $9w$ for $w = 20$. [1.1a]

136. Find the absolute value: $\left|-\frac{4}{13}\right|$. [1.2e]

Write true or false. [1.2d]

137. $-43 < -40$

138. $-3 \geq 0$

139. $-6 \leq -6$

140. $0 > -4$

Synthesis

Determine whether the expressions are equivalent. Explain why if they are. Give an example if they are not. Examples may vary.

141. $3t + 5$ and $3 \cdot 5 + t$

142. $4x$ and $x + 4$

143. $5m + 6$ and $6 + 5m$

144. $(x + y) + z$ and $z + (x + y)$

145. Factor: $q + qr + qrs + qrst$.

146. Collect like terms:

$21x + 44xy + 15y - 16x - 8y - 38xy + 2y + xy$.

1.8 Simplifying Expressions; Order of Operations

We now expand our ability to manipulate expressions by first considering opposites of sums and differences. Then we simplify expressions involving parentheses.

a Opposites of Sums

What happens when we multiply a real number by -1? Consider the following products:

$$-1(7) = -7, \quad -1(-5) = 5, \quad -1(0) = 0.$$

From these examples, it appears that when we multiply a number by -1, we get the opposite, or additive inverse, of that number.

THE PROPERTY OF −1

For any real number a,

$$-1 \cdot a = -a.$$

(Negative one times a is the opposite, or additive inverse, of a.)

The property of -1 enables us to find expressions equivalent to opposites of sums.

EXAMPLES Find an equivalent expression without parentheses.

1. $-(3 + x) = -1(3 + x)$ Using the property of -1

$= -1 \cdot 3 + (-1)x$ Using a distributive law, multiplying each term by -1

$= -3 + (-x)$ Using the property of -1

$= -3 - x$

2. $-(3x + 2y + 4) = -1(3x + 2y + 4)$ Using the property of -1

$= -1(3x) + (-1)(2y) + (-1)4$ Using a distributive law

$= -3x - 2y - 4$ Using the property of -1

Do Exercises 1 and 2.

Suppose we want to remove parentheses in an expression like

$$-(x - 2y + 5).$$

We can first rewrite any subtractions inside the parentheses as additions. Then we take the opposite of each term:

$$-(x - 2y + 5) = -[x + (-2y) + 5]$$
$$= -x + 2y + (-5) = -x + 2y - 5.$$

The most efficient method for removing parentheses is to replace each term in the parentheses with its opposite ("change the sign of every term"). Doing so for $-(x - 2y + 5)$, we obtain $-x + 2y - 5$ as an equivalent expression.

OBJECTIVES

- **a** Find an equivalent expression for an opposite without parentheses, where an expression has several terms.
- **b** Simplify expressions by removing parentheses and collecting like terms.
- **c** Simplify expressions with parentheses inside parentheses.
- **d** Simplify expressions using the rules for order of operations.

SKILL TO REVIEW

Objective 1.6a: Divide integers.

Divide.

1. $20 \div (-4)$ **2.** $-42 \div (-6)$

Find an equivalent expression without parentheses.

1. $-(x + 2)$

2. $-(5x + 2y + 8)$

Answers

Skill to Review:

1. −5 **2.** 7

Margin Exercises:

1. $-x - 2$ **2.** $-5x - 2y - 8$

Find an equivalent expression without parentheses. Try to do this in one step.

3. $-(6 - t)$

4. $-(x - y)$

5. $-(-4a + 3t - 10)$

6. $-(18 - m - 2n + 4z)$

Remove parentheses and simplify.

7. $5x - (3x + 9)$

8. $5y - 2 - (2y - 4)$

Remove parentheses and simplify.

9. $6x - (4x + 7)$

10. $8y - 3 - (5y - 6)$

11. $(2a + 3b - c) - (4a - 5b + 2c)$

Answers

3. $-6 + t$ **4.** $-x + y$ **5.** $4a - 3t + 10$
6. $-18 + m + 2n - 4z$ **7.** $2x - 9$
8. $3y + 2$ **9.** $2x - 7$ **10.** $3y + 3$
11. $-2a + 8b - 3c$

EXAMPLES Find an equivalent expression without parentheses.

3. $-(5 - y) = -5 + y$ Changing the sign of each term

4. $-(2a - 7b - 6) = -2a + 7b + 6$

5. $-(-3x + 4y + z - 7w - 23) = 3x - 4y - z + 7w + 23$

Do Exercises 3–6.

b Removing Parentheses and Simplifying

When a sum is added to another expression, as in $5x + (2x + 3)$, we can simply remove, or drop, the parentheses and collect like terms because of the associative law of addition:

$$5x + (2x + 3) = 5x + 2x + 3 = 7x + 3.$$

On the other hand, when a sum is subtracted from another expression, as in $3x - (4x + 2)$, we cannot simply drop the parentheses. However, we can subtract by adding an opposite. We then remove parentheses by changing the sign of each term inside the parentheses and collecting like terms.

EXAMPLE 6 Remove parentheses and simplify.

$$\begin{aligned} 3x - (4x + 2) &= 3x + [-(4x + 2)] && \text{Adding the opposite of } (4x + 2) \\ &= 3x + (-4x - 2) && \text{Changing the sign of each term inside the parentheses} \\ &= 3x - 4x - 2 \\ &= -x - 2 && \text{Collecting like terms} \end{aligned}$$

Caution!

Note that $3x - (4x + 2) \neq 3x - 4x + 2$. You cannot simply drop the parentheses.

Do Exercises 7 and 8.

In practice, the first three steps of Example 6 are usually combined by changing the sign of each term in parentheses and then collecting like terms.

EXAMPLES Remove parentheses and simplify.

7. $$\begin{aligned} 5y - (3y + 4) &= 5y - 3y - 4 && \text{Removing parentheses by changing the sign of every term inside the parentheses} \\ &= 2y - 4 && \text{Collecting like terms} \end{aligned}$$

8. $$\begin{aligned} 3x - 2 - (5x - 8) &= 3x - 2 - 5x + 8 \\ &= -2x + 6 \end{aligned}$$

9. $$\begin{aligned} &(3a + 4b - 5) - (2a - 7b + 4c - 8) \\ &= 3a + 4b - 5 - 2a + 7b - 4c + 8 \\ &= a + 11b - 4c + 3 \end{aligned}$$

Do Exercises 9–11.

Next, consider subtracting an expression consisting of several terms multiplied by a number other than 1 or -1.

EXAMPLE 10 Remove parentheses and simplify.

$$\begin{aligned} x - 3(x + y) &= x + [-3(x + y)] && \text{Adding the opposite of } 3(x + y) \\ &= x + [-3x - 3y] && \text{Multiplying } x + y \text{ by } -3 \\ &= x - 3x - 3y \\ &= -2x - 3y && \text{Collecting like terms} \end{aligned}$$

EXAMPLES Remove parentheses and simplify

11. $3y - 2(4y - 5) = 3y - 8y + 10$ Multiplying each term in the parentheses by -2

$= -5y + 10$

12. $(2a + 3b - 7) - 4(-5a - 6b + 12)$

$= 2a + 3b - 7 + 20a + 24b - 48 = 22a + 27b - 55$

13. $2y - \frac{1}{3}(9y - 12) = 2y - 3y + 4 = -y + 4$

14. $6(5x - 3y) - 2(8x + y) = 30x - 18y - 16x - 2y = 14x - 20y$

Do Exercises 12–16.

Remove parentheses and simplify.

12. $y - 9(x + y)$

13. $5a - 3(7a - 6)$

14. $4a - b - 6(5a - 7b + 8c)$

15. $5x - \dfrac{1}{4}(8x + 28)$

16. $4.6(5x - 3y) - 5.2(8x + y)$

C Parentheses Within Parentheses

In addition to parentheses, some expressions contain other grouping symbols such as brackets [ ] and braces { }.

> When more than one kind of grouping symbol occurs, do the computations in the innermost ones first. Then work from the inside out.

EXAMPLES Simplify.

15. $[3 - (7 + 3)] = [3 - 10] = -7$

16.
$$\begin{aligned} \{8 - [9 - (12 + 5)]\} &= \{8 - [9 - 17]\} && \text{Computing } 12 + 5 \\ &= \{8 - [-8]\} && \text{Computing } 9 - 17 \\ &= 8 + 8 = 16 \end{aligned}$$

17.
$$\begin{aligned} \left[(-4) \div \left(-\tfrac{1}{4}\right)\right] \div \tfrac{1}{4} &= [(-4) \cdot (-4)] \div \tfrac{1}{4} && \text{Working within the brackets; computing } (-4) \div \left(-\tfrac{1}{4}\right) \\ &= 16 \div \tfrac{1}{4} \\ &= 16 \cdot 4 = 64 \end{aligned}$$

18.
$$\begin{aligned} &4(2 + 3) - \{7 - [4 - (8 + 5)]\} \\ &= 4 \cdot 5 - \{7 - [4 - 13]\} && \text{Working with the innermost parentheses first} \\ &= 20 - \{7 - [-9]\} && \text{Computing } 4 \cdot 5 \text{ and } 4 - 13 \\ &= 20 - 16 && \text{Computing } 7 - [-9] \\ &= 4 \end{aligned}$$

Do Exercises 17–20.

Simplify.

17. $12 - (8 + 2)$

18. $9 - [10 - (13 + 6)]$

19. $[24 \div (-2)] \div (-2)$

20. $5(3 + 4) - \{8 - [5 - (9 + 6)]\}$

Answers

12. $-9x - 8y$ **13.** $-16a + 18$
14. $-26a + 41b - 48c$ **15.** $3x - 7$
16. $-18.6x - 19y$ **17.** 2 **18.** 18
19. 6 **20.** 17

EXAMPLE 19 Simplify.

$$[5(x + 2) - 3x] - [3(y + 2) - 7(y - 3)]$$

$$= [5x + 10 - 3x] - [3y + 6 - 7y + 21]$$ Working with the innermost parentheses first

$$= [2x + 10] - [-4y + 27]$$ Collecting like terms within brackets

$$= 2x + 10 + 4y - 27$$ Removing brackets

$$= 2x + 4y - 17$$ Collecting like terms

21. Simplify:
$[3(x + 2) + 2x] -$
$[4(y + 2) - 3(y - 2)]$.

Do Exercise 21.

d Order of Operations

When several operations are to be done in a calculation or a problem, we apply the following rules.

RULES FOR ORDER OF OPERATIONS

1. Do all calculations within grouping symbols before operations outside.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in order from left to right.
4. Do all additions and subtractions in order from left to right.

These rules are consistent with the way in which most computers and scientific calculators perform calculations.

EXAMPLE 20 Simplify: $-34 \cdot 56 - 17$.

There are no parentheses or powers, so we start with the third step.

$$-34 \cdot 56 - 17 = -1904 - 17$$ Doing all multiplications and divisions in order from left to right

$$= -1921$$ Doing all additions and subtractions in order from left to right

EXAMPLE 21 Simplify: $25 \div (-5) + 50 \div (-2)$.

There are no calculations inside parentheses and no powers. The parentheses with (-5) and (-2) are used only to represent the negative numbers. We begin by doing all multiplications and divisions.

$$\underbrace{25 \div (-5)} + \underbrace{50 \div (-2)}$$

$$= -5 + (-25)$$ Doing all multiplications and divisions in order from left to right

$$= -30$$ Doing all additions and subtractions in order from left to right

Simplify.

22. $23 - 42 \cdot 30$

23. $32 \div 8 \cdot 2$

24. $-24 \div 3 - 48 \div (-4)$

Do Exercises 22–24.

Answers

21. $5x - y - 8$ **22.** -1237 **23.** 8 **24.** 4

EXAMPLE 22 Simplify: $-2^4 + 51 \cdot 4 - (37 + 23 \cdot 2)$.

$$
\begin{aligned}
&-2^4 + 51 \cdot 4 - (37 + 23 \cdot 2) && \\
&= -2^4 + 51 \cdot 4 - (37 + 46) && \text{Following the rules for order of operations within the parentheses first} \\
&= -2^4 + 51 \cdot 4 - 83 && \text{Completing the addition inside parentheses} \\
&= -16 + 51 \cdot 4 - 83 && \text{Evaluating exponential expressions. Note that } -2^4 \neq (-2)^4. \\
&= -16 + 204 - 83 && \text{Doing all multiplications} \\
&= 188 - 83 && \text{Doing all additions and subtractions in order from left to right} \\
&= 105 &&
\end{aligned}
$$

A fraction bar can play the role of a grouping symbol, although such a symbol is not as evident as the others.

EXAMPLE 23 Simplify: $\dfrac{-64 \div (-16) \div (-2)}{2^3 - 3^2}$.

An equivalent expression with brackets as grouping symbols is

$$[-64 \div (-16) \div (-2)] \div [2^3 - 3^2].$$

This shows, in effect, that we do the calculations in the numerator and then in the denominator, and divide the results:

$$\frac{-64 \div (-16) \div (-2)}{2^3 - 3^2} = \frac{4 \div (-2)}{8 - 9} = \frac{-2}{-1} = 2.$$

Do Exercises 25 and 26.

Simplify.

25. $-4^3 + 52 \cdot 5 + 5^3 - (4^2 - 48 \div 4)$

26. $\dfrac{5 - 10 - 5 \cdot 23}{2^3 + 3^2 - 7}$

STUDY TIPS

PREPARING FOR AND TAKING A TEST

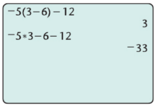
- **Do a thorough review of the chapter, focusing on the objectives and the examples.** Study the notes that you have taken in class also, as well as any hand-outs that your instructor has prepared for you.
- **Do the review exercises in the Summary and Review at the end of the chapter.** Check your answers using the answers at the back of the book. If you have trouble with an exercise, return to the objective indicated by the objective symbol given with the exercise and study that material further.
- **Do the Chapter Test at the end of the chapter.** Check your answers using the answers at the back of the book. Use the objective symbols in the answer section to direct yourself to material that requires further study.
- **When taking a test, read each question carefully. Try to answer all the questions the first time through, but be sure to pace yourself.** Don't allow yourself to spend a disproportionate amount of time on any one question. As you answer the questions, mark those to recheck if you have time.
- **Write your test in a neat and orderly manner.** This will make it easier for you to recheck your work and will also allow your instructor to follow your work when grading your test.

Answers

25. 317 **26.** -12

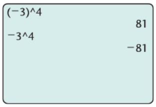
Calculator Corner

Order of Operations and Grouping Symbols Parentheses are necessary in some calculations in order to ensure that operations are performed in the desired order. To simplify $-5(3 - 6) - 12$, we press `(-)` `5` `(` `3` `−` `6` `)` `−` `1` `2` `ENTER`. The result is 3. Without parentheses, the computation is $-5 \cdot 3 - 6 - 12$, and the result is -33.



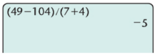
```
-5(3-6)-12
                 3
-5*3-6-12
               -33
```

When a negative number is raised to an even power, parentheses must also be used. To find $(-3)^4$, we press `(` `(-)` `3` `)` `^` `4` `ENTER`. The result is 81. Without parentheses, the computation is $-3^4 = -1 \cdot 3^4 = -1 \cdot 81 = -81$.



```
(-3)^4
                81
-3^4
               -81
```

To simplify an expression like $\dfrac{49 - 104}{7 + 4}$, we must enter it as $(49 - 104) \div (7 + 4)$. We press `(` `4` `9` `−` `1` `0` `4` `)` `÷` `(` `7` `+` `4` `)` `ENTER`. The result is -5.



```
(49-104)/(7+4)
                -5
```

Exercises: Calculate.

1. $-8 + 4(7 - 9) + 5$

2. $-3[2 + (-5)]$

3. $7[4 - (-3)] + 5[3^2 - (-4)]$

4. $(-7)^6$

5. $(-17)^5$

6. $(-104)^3$

7. -7^6

8. -17^5

9. -104^3

10. $\dfrac{38 - 178}{5 + 30}$

11. $\dfrac{311 - 17^2}{2 - 13}$

12. $785 - \dfrac{285 - 5^4}{17 + 3 \cdot 51}$

1.8 Exercise Set

For Extra Help

MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

a Find an equivalent expression without parentheses.

1. $-(2x + 7)$

2. $-(8x + 4)$

3. $-(8 - x)$

4. $-(a - b)$

5. $-(4a - 3b + 7c)$

6. $-(x - 4y - 3z)$

7. $-(6x - 8y + 5)$

8. $-(4x + 9y + 7)$

9. $-(3x - 5y - 6)$

10. $-(6a - 4b - 7)$

11. $-(-8x - 6y - 43)$

12. $-(-2a + 9b - 5c)$

b Remove parentheses and simplify.

13. $9x - (4x + 3)$

14. $4y - (2y + 5)$

15. $2a - (5a - 9)$

16. $12m - (4m - 6)$

17. $2x + 7x - (4x + 6)$

18. $3a + 2a - (4a + 7)$

19. $2x - 4y - 3(7x - 2y)$

20. $3a - 9b - 1(4a - 8b)$

21. $15x - y - 5(3x - 2y + 5z)$

22. $4a - b - 4(5a - 7b + 8c)$

23. $(3x + 2y) - 2(5x - 4y)$

24. $(-6a - b) - 5(2b + a)$

25. $(12a - 3b + 5c) - 5(-5a + 4b - 6c)$

26. $(-8x + 5y - 12) - 6(2x - 4y - 10)$

c Simplify.

27. $9 - 2(5 - 4)$

28. $6 - 5(8 - 4)$

29. $8[7 - 6(4 - 2)]$

30. $10[7 - 4(7 - 5)]$

31. $[4(9 - 6) + 11] - [14 - (6 + 4)]$

32. $[7(8 - 4) + 16] - [15 - (7 + 8)]$

33. $[10(x + 3) - 4] + [2(x - 1) + 6]$

34. $[9(x + 5) - 7] + [4(x - 12) + 9]$

35. $[7(x + 5) - 19] - [4(x - 6) + 10]$

36. $[6(x + 4) - 12] - [5(x - 8) + 14]$

37. $3\{[7(x - 2) + 4] - [2(2x - 5) + 6]\}$

38. $4\{[8(x - 3) + 9] - [4(3x - 2) + 6]\}$

39. $4\{[5(x - 3) + 2] - 3[2(x + 5) - 9]\}$

40. $3\{[6(x - 4) + 5] - 2[5(x + 8) - 3]\}$

d Simplify.

41. $8 - 2 \cdot 3 - 9$

42. $8 - (2 \cdot 3 - 9)$

43. $(8 - 2 \cdot 3) - 9$

44. $(8 - 2)(3 - 9)$

45. $[(-24) \div (-3)] \div \left(-\frac{1}{2}\right)$

46. $[32 \div (-2)] \div \left(-\frac{1}{4}\right)$

47. $16 \cdot (-24) + 50$

48. $10 \cdot 20 - 15 \cdot 24$

49. $2^4 + 2^3 - 10$

50. $40 - 3^2 - 2^3$

51. $5^3 + 26 \cdot 71 - (16 + 25 \cdot 3)$

52. $4^3 + 10 \cdot 20 + 8^2 - 23$

53. $4 \cdot 5 - 2 \cdot 6 + 4$

54. $4 \cdot (6 + 8)/(4 + 3)$

55. $4^3/8$

56. $5^3 - 7^2$

57. $8(-7) + 6(-5)$

58. $10(-5) + 1(-1)$

59. $19 - 5(-3) + 3$

60. $14 - 2(-6) + 7$

61. $9 \div (-3) + 16 \div 8$

62. $-32 - 8 \div 4 - (-2)$

63. $-4^2 + 6$

64. $-5^2 + 7$

65. $-8^2 - 3$

66. $-9^2 - 11$

67. $12 - 20^3$

68. $20 + 4^3 \div (-8)$

69. $2 \cdot 10^3 - 5000$

70. $-7(3^4) + 18$

71. $6[9 - (3 - 4)]$

72. $8[(6 - 13) - 11]$

73. $-1000 \div (-100) \div 10$

74. $256 \div (-32) \div (-4)$

75. $8 - (7 - 9)$

76. $(8 - 7) - 9$

77. $\dfrac{10 - 6^2}{9^2 + 3^2}$

78. $\dfrac{5^2 - 4^3 - 3}{9^2 - 2^2 - 1^5}$

79. $\dfrac{3(6 - 7) - 5 \cdot 4}{6 \cdot 7 - 8(4 - 1)}$

80. $\dfrac{20(8 - 3) - 4(10 - 3)}{10(2 - 6) - 2(5 + 2)}$

81. $\dfrac{|2^3 - 3^2| + |12 \cdot 5|}{-32 \div (-16) \div (-4)}$

82. $\dfrac{|3 - 5|^2 - |7 - 13|}{|12 - 9| + |11 - 14|}$

Skill Maintenance

In each of Exercises 83–90, fill in the blank with the correct term from the given list. Some of the choices may not be used and some may be used more than once.

83. The set of ________________ is $\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. [1.2a]

84. Two numbers whose sum is 0 are called ________________ of each other. [1.3b]

85. The ________________ of addition says that $a + b = b + a$ for any real numbers a and b. [1.7b]

86. The ________________ states that for any real number a, $a \cdot 1 = 1 \cdot a = a$. [1.7a]

87. The ________________ of addition says that $a + (b + c) = (a + b) + c$ for any real numbers a, b, and c. [1.7b]

88. The ________________ of multiplication says that $a(bc) = (ab)c$ for any real numbers a, b, and c. [1.7b]

89. Two numbers whose product is 1 are called ________________ of each other. [1.6b]

90. The equation $y + 0 = y$ illustrates the ________________. [1.7a]

natural numbers
whole numbers
integers
real numbers
multiplicative inverses
additive inverses
commutative law
associative law
distributive law
identity property of 0
identity property of 1
property of -1

Synthesis

Find an equivalent expression by enclosing the last three terms in parentheses preceded by a minus sign.

91. $6y + 2x - 3a + c$

92. $x - y - a - b$

93. $6m + 3n - 5m + 4b$

Simplify.

94. $z - \{2z - [3z - (4z - 5z) - 6z] - 7z\} - 8z$

95. $\{x - [f - (f - x)] + [x - f]\} - 3x$

96. $x - \{x - 1 - [x - 2 - (x - 3 - \{x - 4 - [x - 5 - (x - 6)]\})]\}$

97. Use your calculator to do the following.

a) Evaluate $x^2 + 3$ when $x = 7$, when $x = -7$, and when $x = -5.013$.

b) Evaluate $1 - x^2$ when $x = 5$, when $x = -5$, and when $x = -10.455$.

98. Express $3^3 + 3^3 + 3^3$ as a power of 3.

Find the average.

99. $-15,\ 20,\ 50,\ -82,\ -7,\ -2$

100. $-1,\ 1,\ 2,\ -2,\ 3,\ -8,\ -10$

CHAPTER
1

Summary and Review

Key Terms and Properties

variable, p. 2
constant, p. 2
algebraic expression, p. 3
substitute, p. 3
evaluate, p. 3
natural numbers, p. 9
whole numbers, p. 9
integers, p. 9
opposites, p. 10
rational numbers, p. 11
terminating decimal, p. 13
repeating decimal, p. 13
irrational numbers, p. 13
real numbers, p. 14
absolute value, p. 16
additive inverse, p. 23
reciprocals, p. 46
multiplicative inverse, p. 46
equivalent expressions, p. 54
factor, p. 60
like terms, p. 62
collect like terms, p. 62

Properties of the Real-Number System

The Commutative Laws: $a + b = b + a, \quad ab = ba$

The Associative Laws: $a + (b + c) = (a + b) + c, \quad a(bc) = (ab)c$

The Identity Properties: $a + 0 = 0 + a = a, \quad a \cdot 1 = 1 \cdot a = a$

The Inverse Properties: For any real number a, there is an opposite $-a$ such that $a + (-a) = (-a) + a = 0$.
For any nonzero real number a, there is a reciprocal $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

The Distributive Laws: $a(b + c) = ab + ac, \quad a(b - c) = ab - ac$

The Property of -1: $-1 \cdot a = -a$

Concept Reinforcement

Determine whether each statement is true or false.

________ **1.** Every whole number is also an integer. [1.2d]
________ **2.** The product of an even number of negative numbers is positive. [1.5a]
________ **3.** The product of a number and its multiplicative inverse is -1. [1.6b]
________ **4.** $a < b$ also has the meaning $b \geq a$. [1.2d]

Important Concepts

Objective 1.1a Evaluate algebraic expressions by substitution.

Example Evaluate $y - z$ when $y = 5$ and $z = -7$.

$y - z = 5 - (-7) = 5 + 7 = 12$

Practice Exercise

1. Evaluate $2a + b$ when $a = -1$ and $b = 16$.

Objective 1.2d Determine which of two real numbers is greater and indicate which, using $<$ or $>$.

Example Use $<$ or $>$ for $\square$ to write a true sentence: $-5 \square -12$.

Since -5 is to the right of -12 on the number line, we have $-5 > -12$.

Practice Exercise

2. Use $<$ or $>$ for $\square$ to write a true sentence: $-6 \square -3$.

Objective 1.2e Find the absolute value of a real number.

Example Find the absolute value: **(a)** $|21|$; **(b)** $|-3.2|$; **(c)** $|0|$.

a) The number is positive, so the absolute value is the same as the number.

$|21| = 21$

b) The number is negative, so we make it positive.

$|-3.2| = 3.2$

c) The number is 0, so the absolute value is the same as the number.

$|0| = 0$

Practice Exercise

3. Find: $\left|-\frac{5}{4}\right|$.

Objective 1.3a Add real numbers without using the number line.

Example Add without using the number line: **(a)** $-13 + 4$; **(b)** $-2 + (-3)$.

a) We have a negative number and a positive number. The absolute values are 13 and 4. The difference is 9. The negative number has the larger absolute value, so the answer is negative.

$-13 + 4 = -9$

b) We have two negative numbers. The sum of the absolute values is $2 + 3$, or 5. The answer is negative.

$-2 + (-3) = -5$

Practice Exercise

4. Add without using the number line: $-5.6 + (-2.9)$.

Objective 1.4a Subtract real numbers.

Example Subtract: $-4 - (-6)$.

$-4 - (-6) = -4 + 6 = 2$

Practice Exercise

5. Subtract: $7 - 9$.

Objective 1.5a Multiply real numbers.

Example Multiply: **(a)** $-1.9(4)$; **(b)** $-7(-6)$.

a) The signs are different, so the answer is negative.

$-1.9(4) = -7.6$

b) The signs are the same, so the answer is positive.

$-7(-6) = 42$

Practice Exercise

6. Multiply: $-8(-7)$.

Objective 1.6a Divide integers.

Example Divide: **(a)** $15 \div (-3)$; **(b)** $-72 \div (-9)$.

a) The signs are different, so the answer is negative.

$15 \div (-3) = -5$

b) The signs are the same, so the answer is positive.

$-72 \div (-9) = 8$

Practice Exercise

7. Divide: $-48 \div 6$.

Objective 1.6c Divide real numbers.

Example Divide: **(a)** $-\frac{1}{4} \div \frac{3}{5}$; **(b)** $-22.4 \div (-4)$.

a) We multiply by the reciprocal of the divisor:

$$-\frac{1}{4} \div \frac{3}{5} = -\frac{1}{4} \cdot \frac{5}{3} = -\frac{5}{12}.$$

b) We carry out the long division:

$$-22.4 \div (-4) = 5.6.$$

Practice Exercise

8. Divide: $-\frac{3}{4} \div \left(-\frac{5}{3}\right)$.

Objective 1.7a Simplify fraction expressions.

Example Simplify: $-\frac{18x}{15x}$.

$$\begin{aligned} -\frac{18x}{15x} &= -\frac{6 \cdot 3x}{5 \cdot 3x} && \text{Factoring the numerator and the denominator} \\ &= -\frac{6}{5} \cdot \frac{3x}{3x} && \text{Factoring the fraction expression} \\ &= -\frac{6}{5} \cdot 1 && \frac{3x}{3x} = 1 \\ &= -\frac{6}{5} && \text{Removing a factor of 1} \end{aligned}$$

Practice Exercise

9. Simplify: $\frac{45y}{27y}$.

Objective 1.7c Use the distributive laws to multiply expressions like 8 and $x - y$.

Example Multiply: $3(4x - y + 2z)$.

$$\begin{aligned} &3(4x - y + 2z) \\ &= 3 \cdot 4x - 3 \cdot y + 3 \cdot 2z \\ &= 12x - 3y + 6z \end{aligned}$$

Practice Exercise

10. Multiply: $5(x + 3y - 4z)$.

Objective 1.7d Use the distributive laws to factor expressions like $4x - 12 + 24y$.

Example Factor: $12a - 8b + 4c$.

$$\begin{aligned} &12a - 8b + 4c \\ &= 4 \cdot 3a - 4 \cdot 2b + 4 \cdot c \\ &= 4(3a - 2b + c) \end{aligned}$$

Practice Exercise

11. Factor: $27x + 9y - 36z$.

Objective 1.7e Collect like terms.

Example Collect like terms: $3x - 5y + 8x + y$.

$$\begin{aligned} &3x - 5y + 8x + y \\ &= 3x + 8x - 5y + y \\ &= 3x + 8x - 5y + 1 \cdot y \\ &= (3 + 8)x + (-5 + 1)y \\ &= 11x - 4y \end{aligned}$$

Practice Exercise

12. Collect like terms: $6a - 4b - a + 2b$.

Objective 1.8b Simplify expressions by removing parentheses and collecting like terms.

Example Remove parentheses and simplify:

$5x - 2(3x - y)$.

$5x - 2(3x - y) = 5x - 6x + 2y = -x + 2y$

Practice Exercise

13. Remove parentheses and simplify:

$8a - b - (4a + 3b)$.

Objective 1.8d Simplify expressions using the rules for order of operations.

Example Simplify: $12 - (7 - 3 \cdot 6)$.

$$\begin{aligned} 12 - (7 - 3 \cdot 6) &= 12 - (7 - 18) \\ &= 12 - (-11) \\ &= 12 + 11 \\ &= 23 \end{aligned}$$

Practice Exercise

14. Simplify: $75 \div (-15) + 24 \div 8$.

Review Exercises

The review exercises that follow are for practice. Answers are at the back of the book. If you miss an exercise, restudy the objective indicated in red after the exercise or the direction line that precedes it.

1. Evaluate $\frac{x - y}{3}$ when $x = 17$ and $y = 5$. [1.1a]

2. Translate to an algebraic expression: [1.1b]

Nineteen percent of some number.

3. Tell which integers correspond to this situation: [1.2a]

David has a debt of \$45 and Joe has \$72 in his savings account.

Find the absolute value. [1.2e]

4. $|-38|$

5. $|126|$

Graph the number on the number line. [1.2b]

6. -2.5

7. $\frac{8}{9}$

Use either $<$ or $>$ for $\square$ to write a true sentence. [1.2d]

8. $-3 \; \square \; 10$

9. $-1 \; \square \; -6$

10. $0.126 \; \square \; -12.6$

11. $-\frac{2}{3} \; \square \; -\frac{1}{10}$

12. Write another inequality with the same meaning as $-3 < x$. [1.2d]

Write true or false. [1.2d]

13. $-9 \leq 11$

14. $-11 \geq -3$

Find the opposite. [1.3b]

15. 3.8

16. $-\frac{3}{4}$

Find the reciprocal. [1.6b]

17. $\frac{3}{8}$

18. -7

19. Evaluate $-x$ when $x = -34$. [1.3b]

20. Evaluate $-(-x)$ when $x = 5$. [1.3b]

Compute and simplify.

21. $4 + (-7)$ [1.3a]

22. $6 + (-9) + (-8) + 7$ [1.3a]

23. $-3.8 + 5.1 + (-12) + (-4.3) + 10$ [1.3a]

24. $-3 - (-7) + 7 - 10$ [1.4a]

25. $-\frac{9}{10} - \frac{1}{2}$ [1.4a]

26. $-3.8 - 4.1$ [1.4a]

27. $-9 \cdot (-6)$ [1.5a]

28. $-2.7(3.4)$ [1.5a]

29. $\frac{2}{3} \cdot \left(-\frac{3}{7}\right)$ [1.5a]

30. $3 \cdot (-7) \cdot (-2) \cdot (-5)$ [1.5a]

31. $35 \div (-5)$ [1.6a]

32. $-5.1 \div 1.7$ [1.6c]

33. $-\frac{3}{11} \div \left(-\frac{4}{11}\right)$ [1.6c]

Simplify. [1.8d]

34. $(-3.4 - 12.2) - 8(-7)$

35. $\dfrac{-12(-3) - 2^3 - (-9)(-10)}{3 \cdot 10 + 1}$

36. $-16 \div 4 - 30 \div (-5)$

37. $\dfrac{-4[7 - (10 - 13)]}{|-2(8) - 4|}$

Solve.

38. On the first, second, and third downs, a football team had these gains and losses: 5-yd gain, 12-yd loss, and 15-yd gain, respectively. Find the total gain (or loss). [1.3c]

39. Kaleb's total assets are \$170. He borrows \$300. What are his total assets now? [1.4b]

40. ***Stock Price.*** The value of EFX Corp. stock began the day at \$17.68 per share and dropped \$1.63 per hour for 8 hr. What was the price of the stock after 8 hr? [1.5b]

41. ***Checking Account Balance.*** Yuri had \$68 in his checking account. After writing a check to buy seven equally priced purchases of DVDs, the balance in his account was −\$64.65. What was the price of each DVD? [1.6d]

Multiply. [1.7c]

42. $5(3x - 7)$

43. $-2(4x - 5)$

44. $10(0.4x + 1.5)$

45. $-8(3 - 6x)$

Factor. [1.7d]

46. $2x - 14$

47. $-6x + 6$

48. $5x + 10$

49. $-3x + 12y - 12$

Collect like terms. [1.7e]

50. $11a + 2b - 4a - 5b$

51. $7x - 3y - 9x + 8y$

52. $6x + 3y - x - 4y$

53. $-3a + 9b + 2a - b$

Remove parentheses and simplify.

54. $2a - (5a - 9)$ [1.8b]

55. $3(b + 7) - 5b$ [1.8b]

56. $3[11 - 3(4 - 1)]$ [1.8c]

57. $2[6(y - 4) + 7]$ [1.8c]

58. $[8(x + 4) - 10] - [3(x - 2) + 4]$ [1.8c]

59. $5\{[6(x - 1) + 7] - [3(3x - 4) + 8]\}$ [1.8c]

60. Factor out the greatest common factor: $18x - 6y + 30$. [1.7d]

A. $2(9x - 2y + 15)$ **B.** $3(6x - 2y + 10)$
C. $6(3x + 5)$ **D.** $6(3x - y + 5)$

61. Which expression is *not* equivalent to $mn + 5$? [1.7b]

A. $nm + 5$ **B.** $5n + m$
C. $5 + mn$ **D.** $5 + nm$

Synthesis

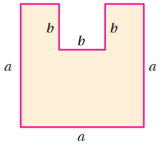
Simplify. [1.2e], [1.4a], [1.6a], [1.8d]

62. $-\left|\frac{7}{8} - \left(-\frac{1}{2}\right) - \frac{3}{4}\right|$

63. $(|2.7 - 3| + 3^2 - |-3|) \div (-3)$

64. $2000 - 1990 + 1980 - 1970 + \cdots + 20 - 10$

65. Find a formula for the perimeter of the figure below. [1.7e]



Understanding Through Discussion and Writing

1. Without actually performing the addition, explain why the sum of all integers from −50 to 50 is 0. [1.3b]

2. What rule have we developed that would tell you the sign of $(-7)^8$ and of $(-7)^{11}$ without doing the computations? Explain. [1.5a]

3. Explain how multiplication can be used to justify why a negative number divided by a negative number is positive. [1.6c]

4. Explain how multiplication can be used to justify why a negative number divided by a positive number is negative. [1.6c]

5. The distributive law was introduced before the discussion on collecting like terms. Why do you think this was done? [1.7c, e]

6. 🖩 Jake keys in $18/2 \cdot 3$ on his calculator and expects the result to be 3. What mistake is he making? [1.8d]

CHAPTER
1

Test

For Extra Help

Chapter Test Prep Videos

Step-by-step test solutions are found on the Chapter Test Prep Videos available via the Video Resources on DVD, in MyMathLab, and on YouTube (search "BittingerIntroInter" and click on "Channels").

1. Evaluate $\frac{3x}{y}$ when $x = 10$ and $y = 5$.

2. Translate to an algebraic expression: Nine less than some number.

Use either $<$ or $>$ for $\square$ to write a true sentence.

3. $-3 \;\square\; -8$

4. $-\frac{1}{2} \;\square\; -\frac{1}{8}$

5. $-0.78 \;\square\; -0.87$

6. Write an inequality with the same meaning as $x < -2$.

7. Write true or false: $-13 \leq -3$.

Simplify.

8. $|-7|$

9. $\left|\frac{9}{4}\right|$

10. $|-2.7|$

Find the opposite.

11. $\frac{2}{3}$

12. -1.4

Find the reciprocal.

13. -2

14. $\frac{4}{7}$

15. Evaluate $-x$ when $x = -8$.

Compute and simplify.

16. $3.1 - (-4.7)$

17. $-8 + 4 + (-7) + 3$

18. $-\frac{1}{5} + \frac{3}{8}$

19. $2 - (-8)$

20. $3.2 - 5.7$

21. $\frac{1}{8} - \left(-\frac{3}{4}\right)$

22. $4 \cdot (-12)$

23. $-\frac{1}{2} \cdot \left(-\frac{3}{8}\right)$

24. $-45 \div 5$

25. $-\frac{3}{5} \div \left(-\frac{4}{5}\right)$

26. $4.864 \div (-0.5)$

27. $-2(16) - |2(-8) - 5^3|$

28. $-20 \div (-5) + 36 \div (-4)$

29. Maureen kept track of the changes in the stock market over a period of 5 weeks. By how many points had the market risen or fallen over this time?

WEEK 1	WEEK 2	WEEK 3	WEEK 4	WEEK 5
Down 13 pts	Down 16 pts	Up 36 pts	Down 11 pts	Up 19 pts

30. ***Antarctica Highs and Lows.*** The continent of Antarctica, which lies in the southern hemisphere, experiences winter in July. The average high temperature is $-67°\text{F}$ and the average low temperature is $-81°\text{F}$. How much higher is the average high than the average low?

Source: National Climatic Data Center

31. ***Population Decrease.*** The population of Mapleton was 18,600. It dropped 420 each year for 6 yr. What was the population of the city after 6 yr?

32. ***Chemical Experiment.*** During a chemical reaction, the temperature in a beaker decreased every minute by the same number of degrees. The temperature was 16°C at 11:08 A.M. By 11:52 A.M., the temperature had dropped to $-17°\text{C}$. By how many degrees did it change each minute?

Multiply.

33. $3(6 - x)$

34. $-5(y - 1)$

Factor.

35. $12 - 22x$

36. $7x + 21 + 14y$

Simplify.

37. $6 + 7 - 4 - (-3)$

38. $5x - (3x - 7)$

39. $4(2a - 3b) + a - 7$

40. $4\{3[5(y - 3) + 9] + 2(y + 8)\}$

41. $256 \div (-16) \div 4$

42. $2^3 - 10[4 - (-2 + 18)3]$

43. Which of the following is *not* a true statement?

A. $-5 \le -5$

B. $-5 < -5$

C. $-5 \ge -5$

D. $-5 = -5$

Synthesis

Simplify.

44. $|-27 - 3(4)| - |-36| + |-12|$

45. $a - \{3a - [4a - (2a - 4a)]\}$

46. Find a formula for the perimeter of the figure shown here.